

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, April-May 2024

B.Tech. (Backlog), Semester VI

Roll No.....

AS1406: Partial Differential Equation and
Fourier Transform

Time: 2.5 hours

Max. Marks: 30

Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. Calculator is not allowed.

- Q.1 Prove the following property of Fourier Cosine transform: 2.0
$$\mathcal{F}_c(f'(x)) = \omega \mathcal{F}_s(f(x)) - f(0),$$
(CO-3)
where $\mathcal{F}_c(f(x)) = \int_0^\infty f(x) \cos(\omega x) dx$.
- Q.2 Classify the following linear second-order differential equations with proper justification: 3.0
a. $2\left(\frac{\partial^2 u}{\partial x^2}\right) + 4\left(\frac{\partial^2 u}{\partial x \partial y}\right) + 3\left(\frac{\partial^2 u}{\partial y^2}\right) = 0.$ (CO-2)
b. $\left(\frac{\partial^2 z}{\partial x^2}\right) - \left(\frac{\partial^2 z}{\partial y^2}\right) = 0.$
- Q.3 Find the Fourier transform of the following function for $k > 0$: $0 < x < \infty$ 3.0
$$f(x) = \begin{cases} \frac{1}{2k}, & \text{if } |x| \leq k \\ 0, & \text{if } |x| > k \end{cases}$$
 (CO-3)
- Q.4 Find a solution of the following P.D.E. by the method of separation of variables: 4.0
 $u_{xx} - u_y = 0$ and $u(0, y) = e^{4y}.$ (CO-1)
- Q.5 Let $f(x) = x$, if $0 < x < \pi$; and $f(x) = 0$ if $x > \pi$. Sketch the graph of this function and find its Fourier cosine transform, $\mathcal{F}_c(f)$. 4.0
(CO-3)
- Q.6 Find the inverse Fourier sine transform $\mathcal{F}_s^{-1}(F(\omega))$ if $F(\omega) = \begin{cases} 1, & 0 \leq \omega < 1 \\ 2, & 1 \leq \omega < 2 \\ 0, & \omega \geq 2 \end{cases}$ 4.0
(CO-3)
- Q.7 Using Fourier Sine transform, solve the Laplace's equation over the region, R: $0 < x < 10$, $0 < y < \infty$, 5.0
(CO-1,2)
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with boundary conditions
$$u(x, 0) = 0, \text{ for } 0 < x < 10,$$
$$u(10, y) = 0, \text{ for } 0 < y < \infty,$$
$$u(0, y) = e^{-y}, \text{ for } 0 < y < \infty.$$
- Q. 8 Find the temperature $u(x, t)$ using method of separation of variables in a laterally insulated heat conducting bar of length 80 cm and diffusivity constant (α^2) as $2 \text{ cm}^2/\text{s}$, if its ends are kept at 0°C and the initial temperature in the bar, $u(x, 0)$, is $100 \sin\left(\frac{\pi x}{80}\right)^\circ\text{C}$. Note that the heat conduction in this bar will be governed by the PDE, $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}.$ 5.0
(CO-2,3)

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