

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Backlog), November 2018

B. Tech (All Branches), Semester III

Roll No.....

COURSE NAME: Engineering Mathematics-III
COURSE CODE: MA 301

Time: 3 hours

Max. Marks: 100

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Assume suitable data wherever required and mention it clearly.
3. Use of calculator is not allowed.

Q.1A Objective type multiple choice questions (10 questions @ 1 mark)

10

- i. The value of $\Gamma\left(\frac{1}{2}\right)$ is
(a) $\sqrt{\pi}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) $2\sqrt{\pi}$ (d) $\frac{\sqrt{\pi}}{2}$
- ii. The Fourier series of an odd function contains only
(a) Terms in sine (b) terms in cosine
(c) only constant terms (d) terms in both sine and cosine
- iii. The value of Bessel function $J_2(x)$ in terms of $J_0(x)$ and $J_1(x)$ is:
(i) $2J_1(x) - xJ_0(x)$ (ii) $\frac{4}{x}J_1(x) - J_0(x)$
(iii) $\frac{2}{x}J_1(x) - \frac{2}{x}J_0(x)$ (iv) $\frac{2}{x}J_1(x) - J_0(x)$
- iv. A Laplace Transform exists when _____
A. The function is piece-wise continuous B. The function is of exponential order
C. The function is piecewise discrete D. The function is of differential order
(a) A & B (b) C & D (c) A & D (d) B & C
- v. Special form of integral transform with kernel $K(s,t)=e^{-st}$ is
(a) Laplace Transform (b) Fourier Sine Transform
(c) Fourier Cosine Transform (d) Complex Fourier Transform
- vi. If $\int_{-1}^{+1} P_n(x)dx = 2$, then n is;
(a) 1 (b) 0 (c) -1 (d) None of theses
- vii. The series $x - \frac{x^3}{2^2(1!)^2} + \frac{x^5}{2^4(2!)^2} - \dots \dots \dots \infty$ equals;
(a) $J_{1/2}(x)$ (b) $J_0(x)$ (c) $xJ_0(x)$ (d) None of theses
- viii. The Laplace transform of $t^2(t)$ is;
(a) $\left(\frac{1}{s^2} + \frac{2}{s}\right)e^{-2s}$ (b) $\frac{1}{s^2}e^{-2s}$ (c) $\left(\frac{1}{s^2} - \frac{2}{s}\right)e^{-2s}$ (d) None of theses
- ix. If f is independent of y then Euler's equation becomes;
(a) $\frac{\partial f}{\partial x} = 0$ (b) $\frac{\partial f}{\partial y} = 0$ (c) $\frac{\partial^2 f}{\partial x^2} = 0$ (d) None of theses
- x. Residue of the function $\frac{e^{2z}}{1-e^z}$ at its pole is ; s

(a) 0 (b) 1 (c) -1 (d) None of these

Q.2 Short answer type questions (4 questions @ 2.5 marks) 10

Q.2A Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

Q.2B Explain Laplace and Fourier Transform.

Q.2C $xP'_n - P'_{n-1} = nP_n$

Q.2D Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at $z=1,2,3$ and infinity and show their sum is zero.

Q.3A Prove that $J_3(x) + 3J_0'(x) + 4J_0'''(x) = 0$ 05

Q.3B Find the Laplace transform of $\frac{s}{s^4 + 4a^4}$. 05

Q.4A Evaluate $\int_c \frac{z^2 - 2z}{(z+1)^2(z^2 + 4)} dz$ where c is the circle $|z|=10$ 05

Q.4B Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$. 05

Q.5A Explain beta, gamma and Bessel function. Also, write their properties. 05

Q.5B Using the Laplace transforms, find the solution of the initial value problem $y'' + 25y = 10 \cos 5t$, $y(0) = 2, y'(0) = 0$. 05

Q.6A Apply method of variation of parameter $(D^2 + 1)y = \operatorname{cosec} x \cdot \cot x$ 05

Q.7A Using Cauchy Integral Formula, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. 05

Q.7B Find Fourier sine and cosine transform of 10

$$f(x) = \begin{cases} x & 0 < x \leq 1 \\ 2-x & 1 < x < 2 \\ 0 & x \geq 2 \end{cases}$$

OR

Q.7B Find the Fourier series of the function $f(x) = \begin{cases} -1 & \text{for } -\pi < x < -\frac{\pi}{2} \\ 0 & \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ +1 & \text{for } \frac{\pi}{2} < x < \pi \end{cases}$ 10

Q.8A Find the Laplace transform of the periodic function Half wave rectifier 10

$$f(t) = \begin{cases} \sin \omega t & \text{for } 0 < t < \frac{\pi}{\omega} \\ 0 & \text{for } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

Q.9A Obtain all possible singularities of $f(z) = \frac{e^z}{(z-1)^2(z-i)(z-3)^3}$, then compute 10

$\oint_{|z|=2} f(z) dz$ using Residue theorem.

Q.9B Using Laplace transform technique solve the following initial value problem 10

$$y'' - 8y' + 12y = f(t); \quad y(0) = -3, \quad y'(0) = 2$$

OR

Q.9B Solve: 10

$$\frac{du}{dx} = \frac{\partial^2 u}{\partial x^2}$$

$$\text{s.t. } u_x(0, t) = 0, \quad \text{and } u(x, 0) = \begin{cases} x, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

$u(x, 0)$ is bounded and $x > 0, t > 0$