

JK Lakshmipat University, Jaipur
Institute of Engineering and Technology
End Term Examination, Nov-Dec 2024
B. Tech., Odd Semester, 2024-25

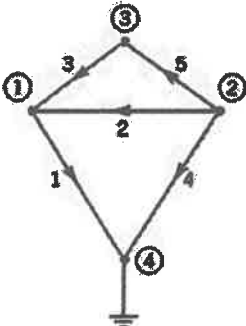
Roll No.

AS1209: Matrix Computations	Time: 2 hours 30 minutes	Max. Marks: 40
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Instructions to students:

- Do not write anything other than your roll number on the question paper.
- Mention all the assumptions for your answers clearly.
- Non-programmable scientific calculator is allowed.

Q. 1	a. Consider a square banded matrix of order 8 with a semi-bandwidth $\beta = 2$. Calculate the degree of sparsity of the matrix. b. Let A be a circulant matrix with the first row $[2 \ 1 \ 3 \ 4]$. Construct the complete 4×4 circulant matrix A. Is A symmetric? c. Given a matrix A of order 9×5, determine the orders of the matrices U, V, and Σ in the singular value decomposition of A. Also, specify the orders of matrices involved in the reduced SVD of A. d. A 5×5 matrix A has singular values 12, 0.1, 0.05, 5 and 0. What is the rank of the matrix? Propose a rank-2 approximation of the matrix based on its singular value decomposition.	4×1.5 (CO 2,3,4)
Q. 2	Consider a small internet with the following structure: Page A links to Pages B, C, and D. Page B links to Page C. Page C links to Pages A and C. Page D links to Pages B and C. Do the following: a. Construct the link matrix L for this internet, where l_{ij} represents the probability of navigating from page j to page i. b. Compute the dominant eigenvector of L, which represents the steady-state PageRank vector. c. Scale the eigenvector such that the sum of its components equals 1. d. Rank the pages (A, B, C, D) based on their PageRank scores and interpret the results.	$1+2+$ $1+1$ (CO 4)
Q. 3	For the matrix $D = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}$. Find the pseudo-inverse of D. Also, find the singular values of D.	2 (CO 4)
Q. 4	a. Compute the QR factorization of the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ 2 & 5 \end{bmatrix}$ using the Householder's method.	$3+2$ (CO 4)

	b. Use the QR factorization of A to find the solution of the linear system $Ax = b$, where $b = \begin{bmatrix} 10 \\ 8 \\ 12 \end{bmatrix}$.	
Q. 5	Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}$. Create a zero in the (3,1) position using the Givens rotation.	2 (CO 4)
Q. 6	For the given matrix: $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 1 \end{bmatrix}$ a. Compute the singular values of A . b. Find the condition number of A with respect to the 2-norm. Also, comment on the conditioning of the matrix.	2+2 (CO 4)
Q. 7	a. Construct an example of a 2×2 matrix with only one distinct eigenvalue. b. Determine if the matrix $A = \begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 7 \\ 0 & 0 & -2 \end{bmatrix}$ is diagonalizable. c. Let A be a 4×4 matrix with eigenvalues 5, 3, and 2, and suppose you know that the eigenspace for 3 is two-dimensional. Do you have enough information to determine if A is diagonalizable? d. Given that the eigenvalues of a matrix A are 1, -1, 3, and 0, what will be the eigenvalues of $(A + 2I)$. Also, what can you say about the determinant of matrix A ? e. Diagonalize the matrix $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$, if possible.	1+1+1+ 1+4 (CO 5)
Q. 8	Given an over-determined linear system $\begin{aligned} 3x + 4y &= 10 \\ 2x + 3y &= 10 \\ 5x + 7y &= 15 \end{aligned}$ Check whether the linear system is consistent. If so, find the solution else find the best possible solution, a vector x , such that $r(x) = \ Ax - b\ _2$ is minimized. How far is Ax from b ?	4 (CO 4)
Q. 9	The network given in the accompanied figure consists of 5 branches/edges and 4 nodes, with one node being grounded. The network can be described by a nodal incidence matrix $A = [a_{ij}]$, where $a_{ij} = \begin{cases} +1, & \text{if branch } j \text{ leaves node } i \\ -1, & \text{if branch } j \text{ enters node } i \\ 0, & \text{if branch } j \text{ doesn't touch } i \end{cases}$ Find the nodal incidence matrix A of the network. Further, find the rank of A .	 4 (CO 1)

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