

JK LakshmiPat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, December 2023

B. Tech. (All branches), Semester V

Roll No.....

AS1202 : Advanced Statistics

Time: 3 hours

Max. Marks: 30

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Use of non-programmable calculators is allowed.
3. Good Luck...

Q1. Carbon dioxide is known to have a critical effect on microbiological growth. Small amount of CO₂ stimulate growth of some organisms, whereas high concentrations inhibit the growth of most. The latter effect is used commercially when perishable food products are stored. A study is conducted to investigate the effect of CO₂ on the growth rate of *Pseudomonas fragi*, a food spoiling organism. Carbon dioxide is administered at five predetermined different atmospheric pressures and three different temperatures. The response measured was the percentage change in cell mass after 1-hour growing time. Five cultures were used at each atmospheric pressure level and temperature, resulting in the following data.

[10]
CO-5

Temp.	0.0	0.083	0.29	0.50	0.86
T1	62.6	50.9	45.5	29.5	24.6
	59.6	44.3	41.1	2.8	17.2
	64.5	47.5	29.8	19.2	7.8
	59.3	49.5	38.3	20.6	10.5
	58.6	48.5	40.2	29.2	17.8
T2	64.6	50.4	38.5	24.1	22.1
	50.9	35.2	30.2	22.6	22.6
	56.2	49.9	27.0	32.7	16.8
	52.3	42.6	40.0	24.4	15.9
	62.1	41.6	33.9	29.6	8.8
T3	59.5	49.5	38.3	20.6	10.5
	38.6	48.6	40.2	29.2	17.8
	44.6	50.4	38.5	24.1	22.1
	50.9	35.2	30.2	22.6	22.6
	29.8	19.2	7.8	14.5	15.3

Construct a statistical model and analyze the data to understand the effect of temperature and pressure on the growth of *fragi* using appropriate statistical technique. Write a report on analysis of the average cell growth.

- Q2. (a) Explain partial correlation and multilinear regression. [2+4]
 (b) An analyst in a certain company was studying the relationship between travel expenses in rupees (y) for 102 sales trips and the duration in day (x) of these trips. He has found that the relationship between y and x is linear. A summary of data is given below: CO-4

$$\sum x = 510, \sum y = 7140, \sum x^2 = 4150, \sum xy = 54900 \text{ and } \sum y^2 = 740200.$$

- (i) Estimate a linear regression line for the above data.
 (ii) Give an approximate 95% confidence interval for the expenses for a 7 day long trip.

- Q3. A private owned liquor store operates both a drive in – facility and a walk-in facility on a randomly selected day, let X and Y, respectively, be the proportions of the time that the drive-in and walk-in facilities are in use, the joint density function of these random variables is [6]
 CO-1,2

$$f(x, y) = \begin{cases} K(x + 2y) & ; \quad 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

- (a) Compute the value of constant K to make it a valid bivariate density.
 (b) Explain the practical significance of marginal densities and calculate the marginal density function for random variable Y.
 (c) Find the probability that the drive-in facility is busy for half of the time.

OR

- (i) Let a random variable X follows binomial distribution. Then derive the expression for its mean and variance. [3+3]
 CO-1, 2,
 3
 (ii) Let X be random variable that has a mean=3 and variance = 4. Let another random variable Y be defined by $Y=3+2X$, then compute the values of $E[3X+Y+8]$, $E[XY]$ and $\text{Var}[(Y-10)/8]$.

- Q4. Explain central limit theorem. How is it important in the study of sum of independent random variables and sampling distributions? [3]
 LO-3
 An auditor takes a random sample of size $n=36$ from a population of 1000 accounts receivable. The mean value of the accounts receivable for the population is Rs. 260 with the population standard deviation Rs 45. What is the probability that the sample mean will be less than Rs 250.

- Q5 Suppose that X and Y are independent random variables following the uniform distribution defined in the intervals $0 \leq x \leq 2$ and $0 \leq y \leq 1$ respectively. Find the probability density function of their sum i.e. find $f_z(z)$ where $Z=X+Y$. Also, find its cumulative density function. [5]
 LO-1, 3

OR

The probability density functions of statistically independent random variables X and Y are

$$f_x(x) = \begin{cases} \frac{2x}{a^2} & ; \quad 0 \leq x < a \\ 0 & ; \text{elsewhere} \end{cases}$$

$$f_y(y) = be^{-by}, y \geq 0 ; \quad a > 0 \text{ and } b > 0 \text{ are constants.}$$

Compute the pdf of random variable $W=X+Y$.