

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Regular & Backlog), November 2024

B. Tech. (CSE), Semester I, 2024-25

Roll No.....

AS1109: Calculus

Time: 3 hours

Max. Marks: 40

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Mention all the assumptions for your answers clearly.
3. Use of calculator is not allowed.

Q. 1	Find the given limits, assuming they exist: a) $\lim_{(x,y) \rightarrow (e,0)} \ln(x^2 + y^2)$ b) $\lim_{(x,y) \rightarrow (1,2)} \frac{(x^2-1)(y^2-4)}{(x-1)(y-2)}$	2 (CO1)
Q. 2	Express $\frac{\partial w}{\partial r}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$.	2 (CO5)
Q. 3	Set up an integral for the arc length of the curve, $y^2 + 2y = 2x + 1$, lying between the points $(-1, -1)$ to $(7, 3)$. Do not evaluate the integral.	2 (CO2, 4)
Q. 4	Find the linearization, $L(x, y, z)$ of the given function at $(0, 0, \frac{\pi}{4})$, $f(x, y, z) = \sqrt{2} \cos(x) \sin(y + z)$.	2 (CO3)
Q. 5	Evaluate $\int_0^1 \int_0^x \int_0^{\ln(x+y)} e^z dz dy dx$.	2 (CO2, 4)
Q. 6	If $\vec{F}(x, y, z) = e^x \hat{i} + yz \hat{j} - yz^2 \hat{k}$, then find the divergence and curl of $\vec{F}$ at the point $(0, 2, -1)$.	2 (CO4)
Q. 7	(a) Sketch the vector field, $F = -x\hat{i}$ and state whether the divergence will be positive or negative. (b) In what direction is the function defined by $f(x, y) = xe^{2y}$ decreasing most rapidly at the point $P_0(2, 0)$?	1.5+1.5 (CO3, 5)
Q. 8	a) Prove that $\text{curl}(\text{grad } \phi) = 0$, where ϕ is a scalar function. b) Using part (a) answer the following as true/ false: (i) $\text{grad } \phi$ is irrotational vector field. (ii) $\text{grad } \phi$ is conservative field.	3 (CO5)
Q. 9	Sketch the region and then find the volume of the solid generated by revolving the region bounded by $y = 2x$, $y = x$ and $x = 1$, about x-axis.	3 (CO2, 4)

Q. 10	Evaluate the given integral by converting it to polar coordinates: $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} x \, dy \, dx.$	3 (CO2, 4)
Q. 11	Sketch the region of integration and evaluate $\int_0^2 \int_{x^2/2}^{\sqrt{2x}} dy \, dx$ by changing the order of integration.	3 (CO2, 4)
Q. 12	An environmental engineer is assessing the impact of two pollutants, x and, y on air quality. The impact I is modeled by: $I(x, y) = 100 + 2x^2 + 3y^2 - 8x - 6y.$ What levels of pollutants x and y minimize the environmental impact? Also, find the value of minimum impact.	3 (CO3)
Q. 13	A developer is assessing a triangular plot of land with vertices at (0,0), (4,0), and (0,4). The area within this triangle will be used for residential buildings. Sketch the triangular plot and calculate the area available for development using double integration.	3 (CO2, 4)
Q. 14	A company is analyzing the temperature distribution in a manufacturing plant, which is represented by the function: $T(x, y) = 100 - 2x^2 - y^2$ where T is the temperature in degrees Celsius, and (x, y) are the coordinates in meters. (a) Find the directional derivative of the temperature function at the point (3, 4) in the direction of the vector $\hat{i} + \sqrt{3}\hat{j}$. (b) Interpret the result in the context of the temperature change in the plant.	3 (CO 5)
Q. 15	For the curve defined by the position vector, $\vec{r}(t) = \cos(3t)\hat{i} - \sin(3t)\hat{j} + 4t\hat{k}$ (a) Find the curvature (b) Find unit normal vector at $t = \frac{\pi}{3}$.	4 (CO 5)

All the best!