

JK Lakshmipat University, Jaipur
Institute of Engineering and Technology
End Term Examination, May 2026
B. Tech., Semester II, 2025-26

Roll No.

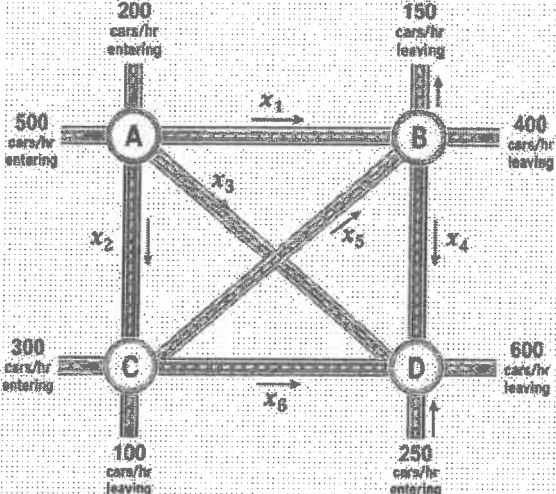
AS1114: Linear Algebra and
Differential Equations

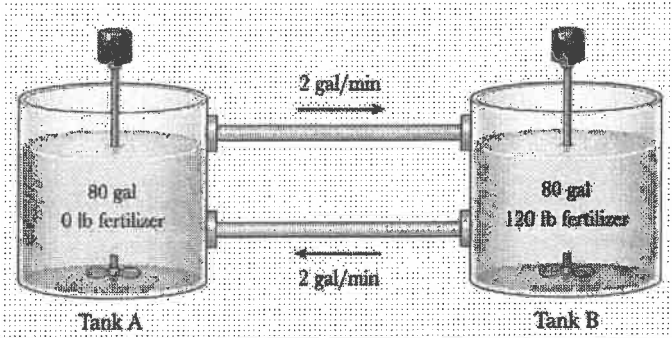
Time: 3 hours

Max. Marks: 40

Instructions to students:

1. Do not write anything, other than your roll number, on the question paper.
2. Provide proper justification for all answers. Answers without adequate reasoning will not be awarded any marks.
3. If a question is attempted in more than one place and none of the attempts is clearly struck off, only the first attempt will be evaluated.
4. Calculator is not allowed.

Q 1	A. Find an ODE for which the following functions form a real basis of solutions. $\{e^{-2x} \cos 3x, e^{-2x} \sin 3x\}$ B. If y_1 is a solution of the given non-homogeneous linear differential equation $y'' - y = \frac{1}{x}, x \neq 0.$ Determine whether $5y_1$ is also a solution of the given differential equation. C. Solve the given ODE: $y^{iv} - 2y'' + y = 0.$	2 (CO 4) 1 (CO 4) 2 (CO 4)
Q 2	The figure shows a traffic network with four intersections A, B, C, and D. The numbers on the external arrows indicate known traffic flows in cars per hour. The variables $x_1, x_2, x_3, x_4, x_5, x_6$ denote the traffic flows on the internal roads, as shown in the diagram.  Assuming that at each intersection, the total inflow equals the total outflow. Write the system of linear equations for the unknown flows. Do not solve the system.	2 (CO 1)
Q 3	A. Find the general solution of the given non-homogeneous linear ODE using the method of variation of parameters. $y'' + 2y' + 10y = e^{-x} \sec(3x).$ B. Solve the given first order linear ODE: $y' + 2y = 4(x + 1)^2.$ C. Solve the Initial Value Problem: $x^2 y'' - xy' - 8y = 0; y(1) = 1, y'(1) = 6.$	4 (CO 4) 3 (CO 4) 3 (CO 4)
Q 4	Are the given functions linearly independent or dependent on the positive x-axis? A. $e^x, e^{-x}, \cosh x$ B. $x^4, 3.5x^n$	2 (CO 4)

Q 5	Two tanks A and B are connected by pipes as shown in the figure. Each tank initially contains 80 gallons of water. Tank A contains no fertilizer, while Tank B contains 120 lb of dissolved fertilizer. Liquid is pumped from Tank A to Tank B at a rate of 2 gal/min and simultaneously from Tank B to Tank A at the same rate of 2 gal/min. The mixtures in both tanks are kept uniform by continuous stirring. Let $y_A(t)$ and $y_B(t)$ (in lb) denote the amounts of fertilizer in Tanks A and B, respectively, at time t (in minutes). Form a system of differential equations satisfied by $y_A(t)$ and $y_B(t)$, along with appropriate initial conditions. 	2 (CO 5)
Q 6	Given A is an $n \times n$ invertible matrix, complete the following statements. A. $\text{rank}(A) =$ _____ B. The system $Ax = b$ has _____ (unique/infinately many/no) solution for every $b \in \mathbb{R}^n$. C. The dimension of the null space of A is _____ D. The number of pivot positions in A is _____ E. $\text{RREF}(A) =$ _____ F. A is similar to a diagonal matrix. (True/False)	3 (CO 1,2)
Q 7	Nilpotent matrix: A square matrix A is called nilpotent if there exists a positive integer k such that $A^k = O$, where O denotes the zero matrix. The smallest such positive integer k is called the index of nilpotency of A. Answer the following questions with justification. A. What can you say about the determinant of a nilpotent matrix? B. If A and B are two commuting nilpotent matrices (i.e., $AB = BA$) with indices of nilpotency k_1 and k_2, respectively, what can you say about the index of nilpotency of the matrix AB?	2 (CO 1,2)
Q 8	Find a basis for the eigenspace corresponding to the eigenvalue, $\lambda = 5$ of the given matrix. $A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}.$ What is the geometric multiplicity and algebraic multiplicity of $\lambda = 5$? Can you conclude about the diagonalizability of the given matrix based on what information you have? Argue. Definition (Eigenspace associated with the eigenvalue λ): The subspace of $\mathbb{R}^n(\mathbb{C}^n)$ consisting of the zero vector and all eigenvectors of A associated with the eigenvalue λ is called the eigenspace associated with λ.	2 (CO 3)
Q 9	A. For a square matrix A, the exponential of A is defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots = \sum_{k=0}^{\infty} \frac{A^k}{k!}.$ Consider the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Compute e^A.	2 (CO 1)

	B. Let A be an invertible square matrix whose characteristic equation is given by $\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0.$ Using the Cayley-Hamilton theorem, express A^{-1} in terms of powers of A.	2 (CO 3)
Q 10	A. Check whether the given ODE is exact. If it is exact find its solution $(\sinh y + y \cosh x)dx + (x \cosh y + \sinh x)dy = 0.$ B. Find an integrating factor of the given ODE: $(\sin y \cos y + x \cos^2 y)dx + x dy = 0.$	3 (CO 4) 2 (CO 4)
Q 11	Let A be a 3×3 matrix with eigenvalues $-2, 3, \frac{1}{4}$ and corresponding eigenvectors $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix},$ respectively. Answer the following: A. Without computation, find matrices P and D such that $A = PDP^{-1}$. B. Write a formula to compute A^7. Also, write the eigenvalues and corresponding eigenvectors of A^7. C. Find the eigenvalues of $2(A - I)^{-1}$.	3 (CO 3)

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