

JK Lakshmipat University, Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY
End Term Examination (Regular/Backlog), November-December 2015
B. Tech (All Branches), Semester I

Roll No.....

MA 101: ENGINEERING MATHEMATICS - I

Time: 3 hours

Max. Marks: 100

Instructions to students:

1. Do not write anything other than your roll number on question paper.

Q.1 Choose the correct answer for the following problems (10 questions @ 1 mark) 10

- i. If $u = x^2 + y^2$ be such that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \lambda u$, then λ is equal to;
(a) 3 (b) 1 (c) 2 (d) none of these
- ii. The percentage error in the area of a rectangle when an error of 1% is made in measuring in length and breadth is equal to;
(a) 1% (b) 2% (c) 0% (d) 3%
- iii. The function $f(x, y) = 2x^2 + 2xy - y^3$ has;
(a) only one stationary point at $(0, 0)$
(b) two stationary points at $(0, 0)$ and $(\frac{1}{6}, \frac{1}{3})$
(c) two stationary points at $(0, 0)$ and $(1, -1)$
(d) None of these
- iv. $x = f(t), y = g(t)$ is called;
(a) Cartesian curve (b) Polar curve (c) Parametric curve (d) none of these
- v. A velocity vector is given as $\vec{V} = 5xy \hat{i} + 2y^2 \hat{j} + 3yz^2 \hat{k}$. The divergence of $\vec{V}$ at $(1, 1, 1)$ is; (a) 9 (b) 10 (c) 14 (d) 15
- vi. The expression $\vec{\nabla} \times (\vec{\nabla} \phi)$, where ϕ is a scalar function, is;
(a) equal to $\nabla^2 \phi$ (b) equal to $\vec{\nabla} \cdot (\vec{\nabla} \phi)$
(c) a scalar of zero magnitude (d) a vector of zero magnitude
- vii. The triple integral $\iiint_R dx dy dz$ represents;
(a) Volume of region R (b) Area of region R
(c) Surface area of region R (d) None of these
- viii. $\int_a^b \pi x^2 dy$ represents;
(a) Volume of the solid of revolution of $x = f(y)$ about y-axis
(b) Volume of the solid of revolution of $y = f(x)$ about y-axis
(c) Volume of the solid of revolution of $x = f(y)$ about x-axis
(d) Volume of the solid of revolution of $y = f(x)$ about x-axis
- ix. In a surface integral $\iint_S \vec{F} \cdot \hat{n} ds$, $\hat{n}$ represents normal vector to;
(a) S (b) $\vec{F}$ (c) either (a) or (b) (d) None of these
- x. The Gauss divergence theorem connects;
(a) a line integral and a surface integral
(b) a surface integral and a volume integral
(c) a line integral and a volume integral
(d) None of these

Q.2 Solve the following problems (4 questions @ 2.5 marks)

i. State Euler's theorem. Prove that:

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -3 \tan u, \text{ given } u = \sin^{-1} \left(\frac{x+2y}{\sqrt{x^8+y^8}} \right).$$

ii. Evaluate $\int_0^{\pi/2} \int_0^{a \cos \theta} r \sin \theta \, dr d\theta$.

iii. Find the unit normal vector to the surface $z = x^2 + y^2$ at the point $(-1, -2, 5)$.

iv. If $\vec{F} = y\hat{i} - x\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ from $(0, 0)$ to $(1, 1)$ along the parabola $y = x^2$.

Q.3A If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, show that $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z} = 0$.

05

Q.3B Discuss the extreme values of $f(x, y) = x^3 - 4xy + 2y^2$.

05

Q.4A Find the asymptotes of the following curve.

05

$$4x^3 - x^2y - 4xy^2 + y^3 + 3x^2 + 2xy - y^2 - 7 = 0.$$

Q.4B Trace the curve $a^2y^2 = x^2(a^2 - x^2)$.

05

Q.5A Trace the curve $r = 2(1 + \cos \theta)$.

05

Q.5B Find the values of a and b so that the surfaces $5x^2 - 2yz - 9z = 0$ may cut the surface $ax^2 + by^3 = 4$ orthogonally at $(1, -1, 2)$.

05

Q.6A Evaluate the greatest rate of increase of $u = xyz^2$ at the point $(1, 0, 3)$?

05

Q.6B If the vector $\vec{A}$ and $\vec{B}$ are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

05

Q.7A Evaluate the integral $I = \int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2} dx dy$ by changing to polar coordinates.

05

Q.7B Evaluate the integral $I = \int_0^1 \int_{x^2}^{2-x} xy dx dy$ by changing the order of integration.

05

Q.8A Find the work done in moving a particle in a force field

05

$\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$.

Q.8B Use Gauss's theorem to prove that $\iint_S \vec{F} \cdot \hat{n} \, ds = \frac{4}{3}\pi(a+b+c)$, given S is the surface of a unit sphere and $\vec{F} = ax\hat{i} + by\hat{j} + cz\hat{k}$; a, b, c are constants.

05

OR

Q.8B In estimating the cost of a pile of bricks measured as $2m \times 15m \times 1.2m$, the tape is stretched 1% beyond the standard length. If the count is 450 bricks to 1 cu. m. and bricks cost Rs. 530 per 1000, find the approximate error in the cost.

05

9A Evaluate $I = \iint_S \vec{F} \cdot \hat{n} \, ds$ where $\vec{F} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ to $z = 5$. 05

Q.9B Use Green's theorem to evaluate $\int_C (xy + y^2)dx + x^2dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 05

OR

Q.9B Prove that $\vec{F} = (y^2 \cos x + z^2)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$ is a conservative field, and find the scalar potential of $\vec{F}$. 05

Q.10 Verify Stoke's theorem for the vector field $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$, integrated around the rectangle $z = 0$ and bounded by the lines $x = 0, y = 0, x = a$ and $y = b$. 10

OR

Q.10 Find the volume and surface of the solid generated by the revolution of the curve: $x = a \cos^3 t, y = a \sin^3 t$ between $t = 0$ and $t = \pi/2$. 10