

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, December 2023

BTech, Semester V (Regular and Backlog)

Roll No.....

AS1406: Partial Differential Equation and
Fourier Transform

Time: 2 hours

Max. Marks: 30

Instructions to students:

- Do not write anything other than your roll number on the question paper.
- Show all the steps properly in each answer.

Q.1	Classify the following linear second-order differential equations. Also, mention the domain of classification: a. $\frac{\partial^2 u}{\partial x^2} + 4 \left(\frac{\partial^2 u}{\partial x \partial y} \right) + 4 \left(\frac{\partial^2 u}{\partial y^2} \right) = 0$ b. $y \left(\frac{\partial^2 z}{\partial x^2} \right) + x \left(\frac{\partial^2 z}{\partial y^2} \right) = 0.$	(3) (CO-2)
Q.2	Show that the Fourier transform is a linear operation; that is, for any functions $f(x)$ and $g(x)$ whose Fourier transforms exist and any constants a and b , the Fourier transform of $af + bg$ exists, and $\mathcal{F}(af + bg) = a\mathcal{F}(f) + b\mathcal{F}(g)$.	(3) (CO-3)
Q.3	Solve the following partial differential equation (PDE) by the method of separation of variables: $3u_x + 2u_y = 0$ and $u(x, 0) = 4e^{-x}$.	(3) (CO-1)
Q.4	Find the Fourier transform of the following function: $f(x) = \begin{cases} -1 & \text{if } -1 < x < 0 \\ 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$	(3) (CO-3)
Q.5	Let $f(x) = \sin(x)$ if $0 < x < \pi$; and $f(x) = 0$ if $x > \pi$. Sketch the graph of this function and find its Fourier sine transform, $\mathcal{F}_s(f)$.	(4) (CO-3)
Q.6	Find the inverse Fourier cosine transform $\mathcal{F}_c^{-1}(F(\omega))$ if $F(\omega) = e^{-\omega}$.	(4) (CO-3)
Q.7	Consider a laterally insulated heat conducting bar of length 2 and diffusivity constant (α^2) as 4, i.e., the heat conduction in this bar will be governed by the following PDE: $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}.$ Find the temperature $u(x, t)$ using method of separation if its ends are kept at 0° and the initial temperature in the bar, $u(x, 0)$ is $\sin\left(\frac{\pi x}{2}\right) + 3 \sin\left(\frac{5\pi x}{2}\right)$.	(5) (CO-1,2)
Q. 8	Using Fourier sine transform, solve the Laplace differential equation over the region, $R: \{(x, y) \mid 0 < x < \infty, 0 < y < 5\},$ $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with boundary conditions $u(0, y) = 0, \text{ for } 0 < y < 5,$ $u(x, 5) = 0, \text{ for } 0 < x < \infty,$ $u(x, 0) = 10, \text{ for } 0 < x < \infty.$	(5) (CO-2,3)

END