

JK Lakshmipat University, Jaipur
Institute of Engineering and Technology
 End Term Examination, December 2025
 B. Tech., Odd Semester, 2025-26

Roll No.

AS1209: Matrix Computations

Time: 2 hours 30 minutes

Max. Marks: 30

Instructions to students:

1. Do not write anything, other than your roll number, on the question paper.
2. Scientific calculator is allowed.

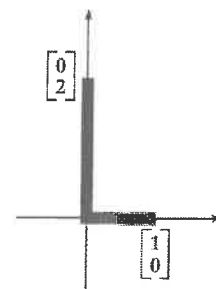
- Q. 1 a. True/False. If $A = [\vec{u} \ \vec{v} \ \vec{w}]$ is any 3×3 matrix, then $\det(A) = \vec{u} \cdot (\vec{v} \times \vec{w})$. Justify. 2 × 3
 b. Can you think of a class of matrices for which eigenvalues are same as singular values? Justify. (Don't answer Identity matrix) (CO 1,4,5)
 c. True/False. Similar matrices have same eigenvalues, rank, determinant, and trace. Justify your answer.

- Q. 2 Consider a small web of four pages A, B, C, and D, with the following links: 1+2
 Page A links to Pages B, and C. (CO 4,5)
 Page B links to Page A, C, and D.
 Page C links to Page A.
 Page D links to Pages A, B, and C.
 Do the following:
 a. Construct the link matrix L for this internet, where l_{ij} represents the probability of navigating from page j to page i .
 b. Compute the dominant eigenvector of L , which represents the steady-state PageRank vector and rank the pages A, B, C, D and interpret the result.

- Q. 3 Let $H = I - \frac{2uu^T}{u^T u}$ be a Householder matrix. Then prove that 3
(CO 4)
 a. $Hu = -u$,
 b. $Hv = v$ if $v^T u = 0$, and
 c. H is symmetric.

- Q. 4 Consider the matrices 2
(CO 1)
 $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0.3 \\ 0 & 1 \end{bmatrix},$ and $D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$

Show the geometric effect of each of these matrices on the standard letter "L" given in the accompanying figure.



- Q. 5 Let $x = \begin{bmatrix} 5 \\ 3 \\ -9 \end{bmatrix}$ and $y = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, find a matrix A of rank one such that $Ax = y$. 2
(CO 1)
 Q. 6 An orthogonal matrix Q always satisfies $\det(Q) = \pm 1$. Explain what this tells about the type of transformation represented by Q in each case. Give an example for each case, and briefly describe its geometric effect. 2
(CO 2,4)

- Q. 7 We are given an almost banded linear system whose coefficient matrix A becomes perfectly banded after creating a zero in the (5,1) position. Use a Givens rotation to annihilate the entry a_{51} . 2
(CO 3,4)

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 \\ 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 2 & 5 & 4 \\ 1 & 0 & 0 & 2 & 1 \end{bmatrix}$$

- Q. 8 Let $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ and the Singular value decomposition of A is given by 2+1+1
(CO 2,4)

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}^T$$

- Find the singular values of A .
- Using the SVD of A , find the SVD of the Moore-Penrose inverse, $A^\dagger$.
- Find the best rank-one approximation of A .

- Q. 9 Given $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix}$. Check whether the linear system $Ax = b$ is consistent. If so, find the solution else find the best possible solution, a vector x , such that $r(x) = \|b - Ax\|_2$ is minimized. How far is Ax from b ? 3
(CO 4)

- Q. 10 Use the Power method to approximate the dominant eigenvalue and dominant eigenvectors of 3
(CO 5)

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Start with $x^{(0)} = [1 \ 1 \ 1]^T$, and perform three iterations of the method.

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