

JK LakshmiPat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, December, 2024

B. Tech. (All branches), Semester V

Roll No.....

AS1202 : Advanced Statistics

Time: 2 hours 30 min

Max. Marks: 30

Instructions to students:

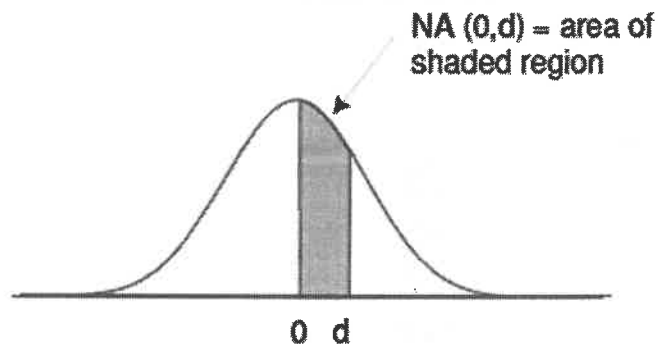
1. Do not write anything other than your roll number on question paper.
2. Use of non-programmable calculators is allowed.

Q1.	Let X be a random variable exponentially distributed with parameter $\lambda > 0$. The probability density function of X is given by: $f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$ If $5E(X) = \text{Var}(X)$, where $E(X)$ and $\text{Var}(X)$ indicate the expectation and variance of X , respectively, the value of λ is _____ (rounded off to one decimal place).	1 CO1
Q2.	Two fair coins are tossed independently. X is a random variable that takes a value of 1 if both tosses are heads and 0 otherwise. Y is a random variable that takes a value of 1 if at least one of the tosses is heads and 0 otherwise. The value of the covariance of X and Y is _____ (rounded off to three decimal places).	1 CO1,2
Q3.	Two correlated random variables, X and Y , have the following characteristics: $\text{Var}(X)=9$, $\text{Var}(Y)=16$, correlation coefficient between X and Y , $\rho = 0.4$. Compute the variance of the sum $Z=X+Y$.	2 CO3
Q4.	In studying the behavior of air support roofs, the random variables X : the inside barometric pressure (in inches of mercury) and Y : the outside pressure is considered. Assume that the joint density function for (X, Y) is given by $f(x, y) = c/x$; $18 \leq Y \leq X \leq 25$, then find the value of constant c which makes it a valid density function. Compute $E(x)$ and $P(X \leq 22 \text{ and } Y \leq 21)$.	3 CO1
Q5.	In a study of size of various computer systems, the random variable X is used for the number of files stored. Past experiences indicates that $\sigma = 5$. The data obtained is as follows: 7 8 4 5 9 9 4 12 8 1 8 7 3 13 2 1 17 7 12 5 6 2 1 13 14 10 2 4 9 11 3 5 12 6 10 7 1. Find an unbiased estimator for μ, the mean number of files per system. 2. Based on Central Limit Theorem, what is the approximate distribution of $\bar{X}$? Write its parameters. 3. What is the probability that the average number of files stored exceeds 10?	3 CO2,3
Q6.	Suppose that X and Y are independent uniform random variables defined in the interval $0 < x < 2$ and $0 < y < 1$. Find the probability density function of their sum i.e. find $f_Z(z)$ where $Z=X+Y$. Also, find its cumulative density function.	5 CO3

Q7.	Since humidity influences evaporation, the solvent balance of water reducible paints during sprayout is affected by humidity. A controlled study is conducted to examine the relationship between humidity (X) and the extent of solvent evaporation(Y). Knowledge of this relationship will be useful in that it will allow the painter to adjust his or her spraygun setting to account for humidity. Summary statistics for these data is as follows- $N=25, \sum x = 1314.90, \sum y = 235.70, \sum x^2 = 76,308.53, \sum y^2 = 2286.07, \sum xy = 11,824.44$ 1. Compute the estimated regression equation for $\hat{\mu}_{y x}$. 2. Predict the extent of solvent evaporation when the relative humidity is 50%. 3. Compute a 95% confidence interval for $\hat{\mu}_{y x}$ when $x=50$ and interpret your result. 4. Compute the value of coefficient of determination for your model and interpret the result.	5 CO4																																																	
Q9	The adhesive selection process is difficult because many factors must be considered, and there is no universal adhesive that will fulfil every application. The efficiency of any adhesive may depend on several external conditions, but an efficient adhesive must effectively work under different atmospheric conditions. Thus whenever an adhesive is manufactures it gets tested for various different parameters. A study is conducted to investigate the effect of temperature and humidity on the force required to separate an adhesive product from a certain material. The experiment is conducted at four specific temperature levels and three specific humidity levels. These levels are not randomly selected. Three measurements are taken at each of these treatment combinations. The resulting data and summary statistics are given below in the table. <table> <tr> <th></th><th colspan="4">Temperature</th></tr> <tr> <th>Humidity</th><th>T1</th><th>T2</th><th>T3</th><th>T4</th></tr> <tr> <td rowspan="3">H1</td><td>40</td><td>39</td><td>32</td><td>33</td></tr> <tr> <td>41</td><td>36</td><td>34</td><td>27</td></tr> <tr> <td>43</td><td>33</td><td>29</td><td>25</td></tr> <tr> <td rowspan="3">H2</td><td>36</td><td>32</td><td>37</td><td>20</td></tr> <tr> <td>34</td><td>29</td><td>23</td><td>22</td></tr> <tr> <td>29</td><td>25</td><td>24</td><td>18</td></tr> <tr> <td rowspan="3">H3</td><td>32</td><td>33</td><td>40</td><td>39</td></tr> <tr> <td>34</td><td>27</td><td>42</td><td>36</td></tr> <tr> <td>29</td><td>25</td><td>43</td><td>33</td></tr> </table> The question to answer statistically is, “Are these differences extreme enough to allow us to conclude that there are real differences in the average responses obtained for these levels of both parameters i.e. temperature and humidity?” (Level of significance : 0.01)		Temperature				Humidity	T1	T2	T3	T4	H1	40	39	32	33	41	36	34	27	43	33	29	25	H2	36	32	37	20	34	29	23	22	29	25	24	18	H3	32	33	40	39	34	27	42	36	29	25	43	33	10 CO5
	Temperature																																																		
Humidity	T1	T2	T3	T4																																															
H1	40	39	32	33																																															
	41	36	34	27																																															
	43	33	29	25																																															
H2	36	32	37	20																																															
	34	29	23	22																																															
	29	25	24	18																																															
H3	32	33	40	39																																															
	34	27	42	36																																															
	29	25	43	33																																															

Best of Luck !

Normal distribution table



	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

F-Critical Distribution Table

The values appearing on this page have been calculated via the built-in proprietary functions of Microsoft Excel. Within each cell, the first entry is the **critical value of F for the 0.05 level of significance**; the second entry (in boldface) is the **critical value for the 0.01 level of significance**.

df denominator	df numerator									
	1	2	3	4	5	6	7	8	9	10
2	18.51 98.50	19.00 99.00	19.16 99.16	19.25 99.25	19.30 99.30	19.33 99.33	19.35 99.36	19.37 99.38	19.38 99.39	19.40 99.40
3	10.13 34.12	9.55 30.82	9.28 29.46	9.12 28.71	9.01 28.24	8.94 27.91	8.89 27.67	8.85 27.49	8.81 27.34	8.79 27.23
4	7.71 21.20	6.94 18.00	6.59 16.69	6.39 15.98	6.26 15.52	6.16 15.21	6.09 14.98	6.04 14.80	6.00 14.66	5.96 14.55
5	6.61 16.26	5.79 13.27	5.41 12.06	5.19 11.39	5.05 10.97	4.95 10.67	4.88 10.46	4.82 10.29	4.77 10.16	4.74 10.05
6	5.99 13.75	5.14 10.92	4.76 9.78	4.53 9.15	4.39 8.75	4.28 8.47	4.21 8.26	4.15 8.10	4.10 7.98	4.06 7.87
7	5.59 12.25	4.74 9.55	4.35 8.45	4.12 7.85	3.97 7.46	3.87 7.19	3.79 6.99	3.73 6.84	3.68 6.72	3.64 6.62
8	5.32 11.26	4.46 8.65	4.07 7.59	3.84 7.01	3.69 6.63	3.58 6.37	3.50 6.18	3.44 6.03	3.39 5.91	3.35 5.81
9	5.12 10.56	4.26 8.02	3.86 6.99	3.63 6.42	3.48 6.06	3.37 5.80	3.29 5.61	3.23 5.47	3.18 5.35	3.14 5.26
10	4.96 10.04	4.10 7.56	3.71 6.55	3.48 5.99	3.33 5.64	3.22 5.39	3.14 5.20	3.07 5.06	3.02 4.94	2.98 4.85
df denominator	df numerator									
	1	2	3	4	5	6	7	8	9	10
11	4.84 9.65	3.98 7.21	3.59 6.22	3.36 5.67	3.20 5.32	3.09 5.07	3.01 4.89	2.95 4.74	2.90 4.63	2.85 4.54
12	4.75 9.33	3.89 6.93	3.49 5.95	3.26 5.41	3.11 5.06	3.00 4.82	2.91 4.64	2.85 4.50	2.80 4.39	2.75 4.30
13	4.67 9.07	3.81 6.70	3.41 5.74	3.18 5.21	3.03 4.86	2.92 4.62	2.83 4.44	2.77 4.30	2.71 4.19	2.67 4.10
14	4.60 8.86	3.74 6.51	3.34 5.56	3.11 5.04	2.96 4.69	2.85 4.46	2.76 4.28	2.70 4.14	2.65 4.03	2.60 3.94
15	4.54 8.68	3.68 6.36	3.29 5.42	3.06 4.89	2.90 4.56	2.79 4.32	2.71 4.14	2.64 4.00	2.59 3.89	2.54 3.80
16	4.49 8.53	3.63 6.23	3.24 5.29	3.01 4.77	2.85 4.44	2.74 4.20	2.66 4.03	2.59 3.89	2.54 3.78	2.49 3.69
17	4.45 8.40	3.59 6.11	3.20 5.19	2.96 4.67	2.81 4.34	2.70 4.10	2.61 3.93	2.55 3.79	2.49 3.68	2.45 3.59