

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Regular), April-May 2016
B. Tech in Computer Science Engineering, Semester IV

Subject Code: CSE402

Roll No.....

Subject Name: Discrete Structures

Time: 3 hours

Max. Marks: 100

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Assume suitable data wherever required and mention it clearly.
3. Writing appropriate units, nomenclature, and drawing neat sketches/schematics/ flow chart wherever, required are an integral part of the answer.

Q.1A Objective type multiple choice questions (10 questions @ 1 mark)

10

- i. Which of the following statement is the negation of the statement, "3 is odd or -5 is negative"?
(a) 3 is even and -5 is not negative (b) 3 is odd and -5 is not negative
(c) 3 is even or -5 is not negative (d) 3 is odd or -5 is not negative
- ii. $q \rightarrow p$ is logically equivalent to
(a) $\neg q \rightarrow p$ (b) $\neg p \rightarrow q$ (c) $\neg p \wedge q$ (d) $\neg q \vee p$
- iii. Let $Q(x, y)$ denote " $x - y = 0$ ". The truth values of the quantifications $\forall x \exists y Q(x, y)$ and $\exists y \forall x Q(x, y)$ are respectively
(a) true, true (b) true, false (c) false, true (d) false, false
- iv. Tautology $[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$ represents ----- rule of inference.
(a) Modus tollens (b) Disjunctive syllogism (c) Simplification (d) Resolution
- v. If e denotes the number of total edges in a graph, then the sum of degrees of all the vertices in a graph is
(a) e (b) $e/2$ (c) e^2 (d) $2e$
- vi. The number of edges in a complete graph K_5 of 5 vertices is equal to
(a) 11 (b) 10 (c) 9 (d) 12
- vii. If $f(x) = \cos x$ and $g(x) = 2x^2 - 1$, then $(g \circ f)(x)$ will be invertible for the domain
(a) $[0, \pi]$ (b) $[0, \frac{\pi}{2}]$ (c) $[\frac{-\pi}{2}, 0]$ (d) $[\frac{-\pi}{4}, \frac{\pi}{4}]$
- viii. If $f: \{1, 2, 3, \dots\} \rightarrow \{0, \pm 1, \pm 2, \dots\}$ is defined by
$$f(x) = \begin{cases} \frac{x}{2}, & \text{if } x \text{ is even} \\ -\frac{(x-1)}{2}, & \text{if } x \text{ is odd} \end{cases}$$

then the value of $f^{-1}(-200)$ is
(a) 200 (b) 399 (c) 400 (d) 401
- ix. Let $R = \{(a, b) \mid a \text{ "divides" } b\}$ be a relation on the set of natural numbers. The relation is
(a) reflexive and transitive (b) reflexive only
(c) an equivalence relation (d) partial order relation
- x. Consider a set A with $|A| = m$. How many possible reflexive relations can be defined on the set A ?



(a) m (b) m^2 (c) 2^{m^2} (d) $2^{m(m-1)}$

Q.2 Short answer type questions (4 questions @ 2.5 marks)

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- Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.
- Is it possible to construct a graph with 12 nodes such that 2 of the nodes have degree 3 and the remaining nodes have degree 4? Justify.
- Shirts numbered consecutively from 1 to 20 are worn by the 20 members of a bowling league. When any 3 of these are chosen to be a team, the sum of their shirt numbers is used as a code number for the team. Show that if any 8 of the 20 are selected then from these 8 we may form at least two different teams having the same code number.
- Let $A = \{1, 2, 3\}$ and let R and S be relations on A whose matrices are given below. Compute $M_{S \circ R}$, where $S \circ R$ denotes the composition of R and S .

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Q.3A Show that $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$ is a tautology.

05

Q.3B Use rules of inferences to show that the hypotheses "Randy works hard", "If Randy works hard, then he is a dull boy", and "If Randy is dull boy, then he will not get the job" imply the conclusion "Randy will not get the job".

05

Q.4A Prove that if n is an integer, these four statements are equivalent:

05

(i) $3x + 2$ is even, (ii) $x + 5$ is odd, (iii) x^2 is even

Q.4B Encrypt the message "WHAT IS YOUR NAME" by translating the letters into numbers, applying the encryption function $f(p) = (p+13) \bmod 26$, and then translating the numbers back into letters.

05

Q.5A Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

05

Q.5B Find $\gcd(9000, 1350)$ and $\text{lcm}(9000, 1350)$ using the prime factorization of integers and verify that $\gcd(9000, 1350) * \text{lcm}(9000, 1350) = 9000 * 1350$.

05

Q.6A Use a loop invariant to prove that the following program segment for computing the n^{th} power, where n is a positive integer, of a real number x is correct.

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```

power := 1
i := 1
while i ≤ n
begin
    power := power * x
    i := i + 1
end

```



Q.6B i. Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?

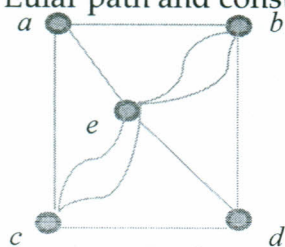
2.5

ii. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph

2.5

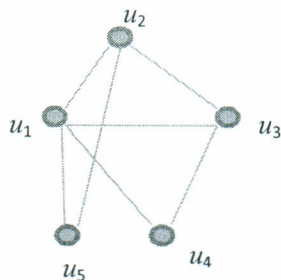


has an Euler path and construct such a path if one exist.

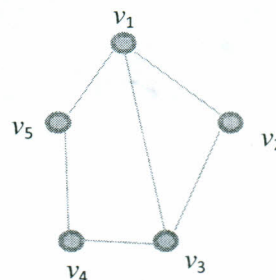


OR

Q.6B Determine whether the graphs G_1 and G_2 shown in figure below are isomorphic. 05

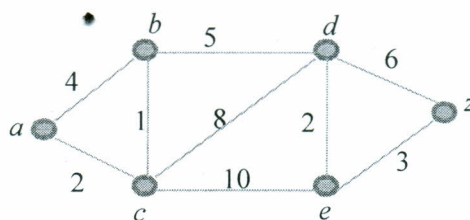


G_1

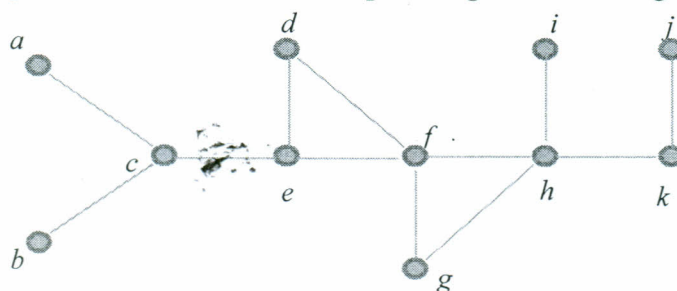


G_2

Q.7A Use Kruskal's algorithm to find a minimum spanning tree in the weighted graph given below: 05



Q.7B Use depth-first search to find a spanning tree for the graph G shown below: 05



OR

Q.7B Suppose that 100 of the 120 mathematics students at a college take at least one of the languages French, German and Russian. Also suppose that 65 study French, 45 study German, 42 study Russian, 20 study French and German, 25 study French and Russian and 15 study German and Russian. 05

- Find the number of students who study all three languages.
- Fill in the correct number of students in each of the eight regions of Venn diagram.
- Determine the number of students who study exactly two languages.

- Q.8A i. Let A, B, C be subsets of the universal set U . Given that $A \cap B = A \cap C$ and $\bar{A} \cap B = \bar{A} \cap C$.
Is it necessary that $B = C$? Justify your answer with an example. 05
- ii. Show that function $f(x) = ax + b$ from R to R is invertible. Where a and b are constants with $a \neq 0$. 05
- Q.9A i. Let $A = B = C = R$ and Let $f: A \rightarrow B, g: B \rightarrow C$ be defined by $f(x) = x + 1$ and $g(x) = x^2 + 2$. Compute the following: 05
- (a) $(f \circ g)(-2)$ (b) $(g \circ f)(-1)$ (c) $(f \circ f)(2)$ (d) $(g \circ g)(-1)$
- ii. Let $R_1 = \{(1, 2), (2, 3), (3, 4)\}$ and $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ be relations from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Compute the following: 05
- (a) $R_1 \cup R_2$ (b) $R_1 \cap R_2$ (c) $R_1 - R_2$ (d) $R_2 - R_1$ (e) R_1^{-1}
- Q.9B i. Determine the transitive closure R^+ and reflexive-transitive closure R^* for the relation $R = \{(1, 1), (2, 1), (3, 2), (4, 3)\}$ on the set $S = \{1, 2, 3, 4\}$. 05
- ii. Let m be a positive integer greater than 1. Consider the relation $R = \{(a, b) \mid a \equiv b \pmod{m}\}$, where $a \equiv b \pmod{m}$ means a and b are congruence modulo m that is a and b leave same remainder when divided by m . In other words $a - b$ is divisible by m . Determine whether or not the relation R is an equivalence relation. 05
- OR
- Q.9B Use Dijkstra's algorithm to find the length of shortest path between the vertices a and z in the weighted graph given in Q.7(A). 10

