

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Makeup Examination (Regular), December 2024
 B. Tech. (CSE), Semester I, 2024-25

Roll No.....

As1109: Calculus

Time: 3 hours

Max. Marks: 40

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Mention all the assumptions for your answers clearly.

Q. 1	By considering different paths of approach, show that the limit of the function $f(x, y) = \frac{y}{x^2 - y}$ does not exist as $(x, y) \rightarrow (0, 0)$.	2 (CO1)
Q. 2	Prove that $y = f(x + at) + g(x - at)$ satisfies $\frac{\partial^2 y}{\partial t^2} = a^2 \left(\frac{\partial^2 y}{\partial x^2} \right)$	2 (CO5)
Q. 3	Evaluate $\iiint_V e^{x+y+z} dx dy dz$, where V is the volume bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=a$.	2 (CO2, 4)
Q. 4	Prove that $xz_x + yz_y = \frac{5}{2} \tan z$ if $z = \sin^{-1} \left(\frac{x^3 + y^3}{\sqrt{x} + \sqrt{y}} \right)$ with the help of Euler's theorem.	2 (CO3)
Q. 5	Change the Cartesian integral $\int_0^6 \int_0^y x dx dy$ into an equivalent polar integral. Then evaluate the polar integral.	2 (CO2, 4)
Q. 6	Show that the vector field defined by $\vec{V} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}$ is irrotational.	2 (CO4)
Q. 7	(a) For the vector field $\vec{v} = [x^2 y^3 - z^4, 4x^5 y^2 z, -y^4 z^6]$, compute $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{v})$. (b) Find the directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$.	1.5+1.5 (CO3, 5)
Q. 8	Suppose $V(x, y, z) = 5x^2 - 3xy + xyz$ represents the electrical potential over a certain region of space. i. Find the rate of change of the potential at $P(3, 4, 5)$ in the direction of the vector $\vec{v} = \hat{i} + \hat{j} - \hat{k}$. ii. In which direction does V change most rapidly at P? iii. What is the maximum rate of change at P?	3 (CO5)
Q. 9	Let $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$. Is it possible to define $f(0, 0)$ in a way that makes f continuous at the origin? Why?	3 (CO2, 4)
Q. 10	Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ from $y = 1$ to $y = 3$.	3 (CO2, 4)

Q. 11	Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line $x = -1$. Draw proper figure to support your answer.	3 (CO2, 4)
Q. 12	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{-3}{(x+y+z)^2}$	3 (CO3)
Q. 13	Find $\nabla \phi$ if (a) $\phi = \log r $ and (b) $\phi = \frac{1}{r}$, where 'r' is the position vector.	1.5+1.5 (CO5)
Q. 14	Explain curvature and unit normal vector. Then, find the radius of curvature $y^2 = 2x(3 - x^2)$ at the points where tangents are parallel to x-axis.	1+2 (CO 5)
Q. 15	Sketch the region of integration and write an equivalent integral with order of integration reversed. Then evaluate both integrals. $\int_0^1 \int_0^{9-4x^2} 16x \, dy \, dx$	4 (CO2, 4)

all the best!