

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Backlog), November 2017

B. Tech.(All Branches), Semester III

Roll No.....

COURSE CODE: MA 301

Time: 3 hours

Max. Marks: 100

COURSE TITLE: Engineering Mathematics III

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Calculators are allowed.

Q.1A Objective type multiple choice questions (10 questions @ 1 mark)

10

- i. Function $f(z) = \sin\left(\frac{1}{z}\right)$ has type of singularity at $z=0$
 (a) removable (b) essential (c) pole (d) zero
- ii. Which of the following is true for $f(z) = z^4 + z$
 (a) Analytic everywhere (b) nowhere analytic
 (c) Analytic only at Origin (d) Analytic everywhere except at Origin
- iii. The value of $J_{-1/2}(x)$ is
 (a) $\sqrt{\frac{2}{\pi x}} \sin x$ (b) $\sqrt{\frac{2}{\pi x}} \cos x$ (c) $\sqrt{\frac{2}{\pi x}} \sin x$ (d) $\sqrt{\frac{2}{\pi x}} \cos x$
- iv. The Laplace transform of $f(t) = t e^t$ is
 (a) $\frac{1}{(s+1)^2}$ (b) $\frac{-1}{(s+1)^2}$ (c) $\frac{-1}{(s-1)^2}$ (d) $\frac{1}{(s-1)^2}$
- v. The value of $J_3(x) + 3J_0'(x) + 4J_0'''(x) = ?$ is :
 (a) $2J_1(x) - xJ_0(x)$ (b) 0
 (c) $\frac{2}{x}J_1(x) - \frac{2}{x}J_0(x)$ (d) 1
- vi. The inverse Laplace transform of $\frac{3}{(s^2+9)}$
 (a) $\sin 3t$ (b) $\cos 3t$ (c) $\sinh 3t$ (d) $\cosh 3t$
- vii. The residue of the function $f(z) = \frac{\cos z}{z}$ about its pole $z=0$, is
 (a) 1 (b) 2 (c) $4/3$ (d) zero
- viii. The value of $\Gamma\left(\frac{3}{2}\right)$ is

	(a) $\sqrt{\pi}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) $2\sqrt{\pi}$ (d) $\frac{\sqrt{\pi}}{2}$	
ix	The value of $\beta\left(\frac{5}{2}, \frac{1}{2}\right)$ is	
	(b) $\frac{\sqrt{\pi}}{16}$ (b) $\frac{\pi}{16}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{8}$	
x	The Laplace transform of $\frac{1}{\sqrt{t}}$ is	
	(a) not defined (b) $\sqrt{\frac{\pi}{s}}$ (c) $\frac{\pi}{\sqrt{s}}$ (d) $\left(\frac{1}{2}\right)! s^{-\frac{1}{2}}$	

Q.2	Short answer type questions (4 questions @ 2.5 marks)	10
(i)	Find the Value of $J_{-3/2}(x)$ in terms of $\sin x$ and $\cos x$.	
(ii)	Explain convolution theorem for Laplace transform.	
(iii)	Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$.	
(iv)	Find the value of $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta$.	
Q.3A	Find the Laplace transform of the periodic function $f(t) = t^2, 0 < t < a$ and $f(t+a) = f(t)$	05
Q.3B	Show that $u = -\sin x \sinh y$ is harmonic. Find the harmonic conjugate of u and hence form the analytic function $f(u,v)$.	05
Q.4A	Find the Laplace transform of $f(t) = \begin{cases} 2 & , 0 < t < \pi \\ 0 & , \pi < t < 2\pi \\ \sin t & , t > 2\pi \end{cases}$.	05
Q.4B	Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 \theta \cos^7 \theta d\theta$.	05
Q.5	Using Residue theorem, evaluate $\int_c \frac{z^2 + 4}{(z^3 + 2z^2 + 2z)} dz$, where c is the circle $ z = 6$.	10
Q.6A	Prove $\frac{d}{dx} [x^p J_p(x)] = x^p J_{p-1}(x)$	05

Q.6B	Evaluate $\int_0^1 x^4 \left[\ln \left(\frac{1}{x} \right) \right]^3 dx$.	05
OR		
Q.6B	Explain convolution theorem for Laplace transform. Hence find inverse Laplace transform of $\frac{1}{s(s+a)^2}$.	05
Q.7A	Obtain the Fourier sine series of $f(x) = \cos x$ in the interval $0 < x < \pi$.	05
Q.7B	Evaluate the following integral using Cauchy integral formula $\int_c \frac{e^z}{z(1-z)^3} dz$, where c is the circle $ z-1 = \frac{1}{2}$.	05
OR		
Q.7B	Solve the following initial value problem using Laplace transform $y'' + 2y' + 5y = e^{-t} \cos t$, $y(0) = 0$, $y'(0) = 0$	05
Q.8A	Using contour integration, evaluate the real integral $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos \theta} d\theta$.	10
Q.9A	Prove that $\int_0^a x J_n(\alpha x) J_n(\beta x) dx = \begin{cases} 0 & ; \text{ if } \alpha \neq \beta \\ \frac{\alpha^2}{2} J_{n+1}^2(a\alpha); \text{ if } \alpha = \beta \end{cases}$ where α and β are roots of $J_n(\alpha x) = 0$.	10
Q.9B	Solve the following system of simultaneous differential equations using Laplace transform $\frac{dx}{dt} - y = e^t$, $\frac{dy}{dt} + x = \sin t$; $x(0) = 1$, $y(0) = 0$.	10
OR		
Q.9B	Find the temperature distribution in semi-infinite bar with its end points and lateral surface insulated and with initial temperature distribution in the bar is prescribed by $f(x)$. Deduce the solution when $f(x) = e^{-ax}$.	10