

JK Lakshmipat University, Jaipur

Institute of Engineering and Technology

End Term Examination

B.Tech. Second year / Third Sem / 2013-2014

Course Name: Engineering Mathematics III

Max Marks: 40

Course Code: MA 301

Time: 03.00 hr

Instructions: 1. Attempt all sections.
2. Calculators are not allowed.

Section A

All questions are compulsory.

- Q1. (i)** Which of the following is true for $f(z) = xy + iy$ (5 × 1 Marks)
- (a) Analytic everywhere (b) nowhere analytic
(b) Analytic only at Origin (d) Analytic everywhere except at Origin
- (ii)** $\int_C \frac{1}{(z^2 + 9)} dz$, $C: |z - 3i| = \frac{1}{2}$ = ...
- (iii)** $J_0 - 2J_2 + 2J_4 - \dots = \dots$
- (iv)** Find $L\{g(t), s\}$, where $g(t) = \begin{cases} 0 & , 0 < t < 5 \\ t - 3 & ; t > 5 \end{cases}$
- (v)** Find $f(x)$ if its F.C.T. is $\frac{1}{s^2 + 1}$ and F.S.T. is $\frac{s}{s^2 + 1}$.
- Q2. (i)** Give the Euler's equation with reference to calculus of variation. (1)
- (ii)** Determine and classify all singularities of $f(z) = \frac{1 - e^z}{z^1 (z^2 + 4)^2}$. (2)
- (iii)** Find the inverse Laplace transform of $F(s) = \frac{1}{s^2 (s - 1)}$ (2)
by convolution theorem.

Section B

(4 × 5 Marks)

Attempt any four questions. All questions carry equal marks.

- Q3.** Show that under the transformation $w = \frac{z - i}{z + i}$, real axis in the z -plane is mapped into the circle $|w| = 1$.
- Q4.** Find the DFT of the 6-point sequence $a(n)$, where $a(n) = \{1, 2, 3, 4\}$. Here $a(0)=1$, $a(1)=2$, $a(2)=3$, $a(3)=4$, $a(4)=5$, $a(5)=6$.

Q5. Solve

$$\frac{du}{dx} = \frac{\partial^2 u}{\partial x^2}$$

$$\text{s.t. } u_x(0, t) = 0, \quad \text{and } u(x, 0) = \begin{cases} x, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

$u(x, 0)$ is bounded and $x > 0, t > 0$

Q6. Evaluate $\int_c \frac{z^2 - 2z}{(z+1)^2(z^2+4)} dz$ where c is the circle $|z|=10$

Q7. Establish the following

$$(i) \quad J_3(x) + 3J_0'(x) + 4J_0'''(x) = 0$$

$$(ii) \quad P_n'(x) - xP_{n-1}'(x) = nP_{n-1}(x)$$

Section C**(2 × 5 Marks)**

Q8. Evaluate $\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2}$ (using complex integration)

OR

Find the Taylor's expansion of $f(z) = \frac{1}{z^2 - z - 6}$ about (a) $z=-1$, (b) $z=1$.

Q9. Using Laplace transform technique solve the following initial value problem

$$y'' - 8y' + 12y = f(t); \quad y(0) = -3, \quad y'(0) = 2$$

OR

A resistance R in series with inductance L is connected with e.m.f. $E(t)$.

The current I is given by $L \frac{di}{dt} + Ri = E(t)$. If the switch is connected at $t=0$ and disconnected at $t=a$, find the current i in terms of t .