

JK Lakshmipat University Jaipur
Institute of Engineering and Technology
 End Term Examination, April 2024
 B. Tech., Semester II, 2023-24

Roll No.....

AS1114: LINEAR ALGEBRA AND
 DIFFERENTIAL EQUATIONS

Time: 3 hours

Max. Marks: 40

Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. Mention all the assumptions for your answers clearly.
3. Calculator is not allowed.

Q. 1.1 Given that a real square matrix Q is said to be orthogonal if $Q^T = Q^{-1}$. What is the determinant of an orthogonal matrix? Explain. 1.0
(CO 1)

Q. 1.2 Let A be a 4×4 matrix with eigenvalues 3, 1 and 2, and suppose that the eigenspace for $\lambda = 2$ is two-dimensional. Do we have enough information to conclude if A is diagonalizable? If yes, state the criteria for diagonalizability. If not, what additional information is required to reach a conclusion? 1.0
(CO 2)

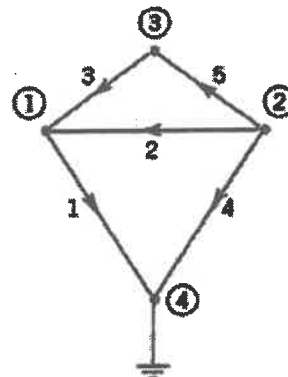
Q. 1.3 Consider a matrix A whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Comment as True/False, along with a justification, on the following statements: 2.0
(CO 1,2)

- a. A cannot be diagonalized.
- b. A has a repeated eigenvalue.

Q. 2.1 The network given in the accompanied figure consists of 5 branches/edges and 4 nodes, with one node being grounded. The network can be described by a **nodal incidence matrix** $A = [a_{ij}]$, where

$$a_{ij} = \begin{cases} +1, & \text{if branch } j \text{ leaves node } i \\ -1, & \text{if branch } j \text{ enters node } i \\ 0, & \text{if branch } j \text{ doesn't touch } i \end{cases}$$

Find the nodal incidence matrix A of the network.
 Further, find the rank of A .

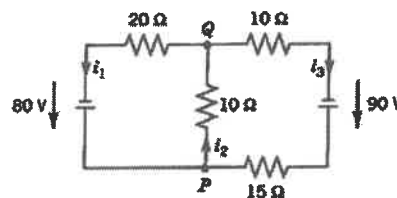


2.0
(CO 1)

Q. 2.2 Following is the system of equations of current flow on applying Kirchhoff's laws to the electric network shown in the figure: 3.0
(CO 1,2)

$$\begin{aligned} i_1 - i_2 + i_3 &= 0 \\ -i_1 + i_2 - i_3 &= 0 \\ 10i_2 + 25i_3 &= 90 \\ 20i_1 + 10i_2 &= 80. \end{aligned}$$

Solve the equations for the currents i_1 , i_2 , and i_3 .



Q. 3.1 Check if the given transformation is linear or not.

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ defined by } T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} |xy| \\ y \\ x+y \end{bmatrix}.$$

1.0
(CO 1)

Q. 3.2	Let $L: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation defined by $L \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{bmatrix} u_1 + u_2 \\ u_3 + u_4 \\ u_1 + u_3 \end{bmatrix}$, Find a basis of kernel and range of linear transformation.	4.0 (CO 1)
Q. 4.1	Find the order and degree of the differential equation, $\left(1 + \frac{dy}{dx}\right)^{2/3} = \frac{d^2y}{dx^2}$.	1.0 (CO 3)
Q. 4.2	Sketch the direction fields for the differential equation, $\frac{dy}{dx} = -\sqrt{3}$.	1.0 (CO 3)
Q. 4.3	For what value(s) of a , the differential equation given below is exact? $(4x^2 + ay + y) dx + (2a^2x - 3y^2 + 1) dy = 0$.	1.0 (CO 3)
Q. 4.4	Find a suitable integrating factor for the differential equation, $(y \ln y + 2xy^2)dx + (x + x^2y)dy = 0$, and use it to find the general solution.	3.0 (CO 3)
Q. 5.1	Given that y_1 and y_2 are solutions of the non-linear ODE, $y' + y^2 = 0$, then what can you say about $k_1y_1 + k_2y_2$, where k_1 and k_2 are real numbers?	1.0 (CO 3)
Q. 5.2	Use Wronskian to check the linear independence of the given functions on the x -axis. $\cosh x, \sinh x, \cosh^2 x$.	2.0 (CO 3)
Q. 5.3	Find an ODE for which the following functions form a basis of solutions: $\cos 2x, \sin 2x, \cos 3x, \sin 3x$.	2.0 (CO 4)
Q. 5.4	Solve the initial value problem (IVP), $y''' - 2y'' + 4y' - 8y = 0, y(0) = -1, y'(0) = 30, y''(0) = 28$.	3.0 (CO 3)
Q. 6.1	Solve the non-homogeneous differential equation by variation of parameters. Give a general solution. $y'' - 2y' + y = x^{-3}e^x$	3.0 (CO 3)
Q. 6.2	Find a general solution of the differential equation by the method of undetermined coefficients. $y'' - 4y = -5e^{3x} + 8x$	4.0 (CO 3)
Q. 7.1	Solve the following system of ODEs with given initial conditions using the concept of eigenvalues: $y_1' = 3y_1 - 4y_2,$ $y_2' = 2y_1 - 3y_2,$ $y_1(0) = 7, y_2(0) = 5$.	4.0 (CO 2,3)
Q. 7.2	Consider the matrix $A = \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}$ obtained in the question 7.1, determine an invertible matrix P and a diagonal matrix D such that A is similar to D .	1.0 (CO 1,2)

∴∴END∴∴