

JK Lakshmipat University Jaipur
Institute of Engineering and Technology

End Term Examination, May 2025

B. Tech., Semester II, 2024-25

Roll No.....

AS1114: LINEAR ALGEBRA AND
DIFFERENTIAL EQUATIONS

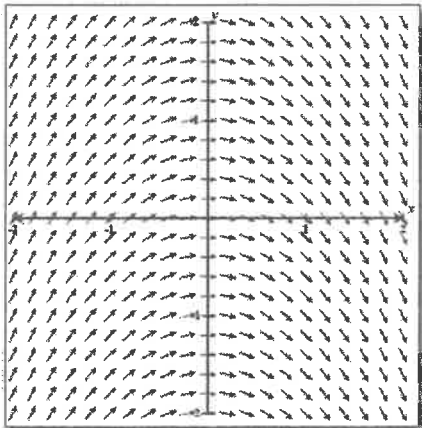
Time: 2.5 hours

Max. Marks: 30

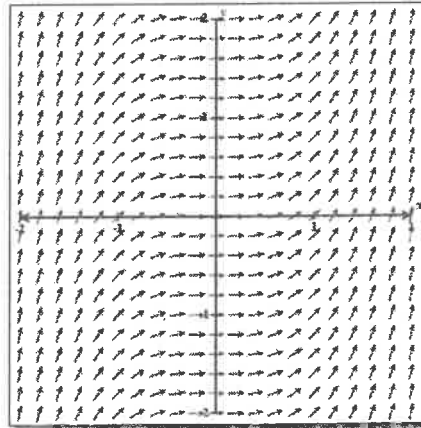
Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. Mention all the assumptions for your answers clearly.
3. Calculator is not allowed.

- Q 1.1 The figures below show direction fields for the differential equation (DE), $\frac{dy}{dx} = f(x, y)$. 1.0
(CO 4)
For each figure, identify the corresponding function $f(x, y)$.



a)



b)

- Q 1.2 Check the given differential equation for exactness and hence, solve. 2.0
(CO 4)
 $\sinh x \sinh y \, dx + \cosh x \cosh y \, dy = 0$.

- Q 1.3 Find two integrating factors for the differential equation: 2.0
(CO 4)
 $(2xy^2)dx + (3x^2y)dy = 0$
such that one integrating factor should be a function of x only and other should be a function of y only. Do not manipulate the DE. No need to solve the DE.

- Q 2.1 Transform the given non-linear ODE (Bernoulli's equation) to a linear ODE and hence, solve it. 3.0
(CO 4)

$$y' + x^2y = \frac{e^{-x^3} \sin x}{3y^2}.$$

- Q 2.2 Solve the non-homogeneous differential equation. Give a general solution. 4.0
(CO 4)
 $x^2y'' + 6xy' + 6y = x^2$

Q 3.1 Given that y_1 and y_2 are solutions of a non-homogeneous linear differential equation, is $3y_1 - y_2$ also a solution? 1.0
(CO 4)

Q 3.2 Show the linear independence of the given functions on the x -axis using Wronskian. 2.0
 $1, x, \sin x, \cos x.$ (CO 1,4)

Q 3.3 Find the general solution of the given ODE: 2.0
 $y^{iv} + 8y'' + 16y = 0.$ (CO 4)

Q 4.1 If V is a vector space that has a nonzero vector, how many vectors are in V ? 1.0
(CO 2)

Q 4.2 Suppose a square matrix A is reduced to an echelon form B using a sequence of row replacements and row interchanges. Given that A is an invertible, how is the $\det(B)$ related to $\det(A)$? Explain your reasoning. 1.0
(CO 1)

Q 4.3 Given A be a 4×9 non-zero matrix which represents the standard matrix of a linear transformation $T: \mathbb{R}^{k_1} \rightarrow \mathbb{R}^{k_2}$. 2.0
(CO 1,2)
a) What are the values of k_1 and k_2 ?
b) $\underline{\hspace{2cm}} \leq \dim(\text{range } T) \leq \underline{\hspace{2cm}}$
c) Are the columns of A linearly dependent or linearly independent?

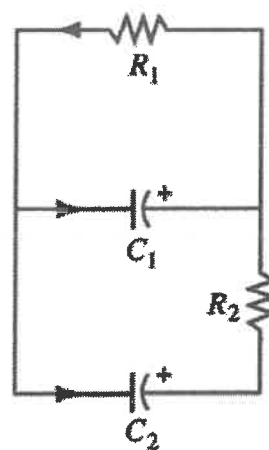
Q 5.1 If A is a 3×3 matrix with eigenvalues 2, -2 and 3, can we say that A is diagonalizable? 1.0
Further, can you comment on the diagonalizability of A^2 ? (CO 3)

Q 5.2 Given $A = \begin{bmatrix} 5 & -8 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & -2 \end{bmatrix}$, find A^{-1} using Cayley-Hamilton theorem. Hence, find the 3.0
value of $A^8 - 4A^7 - 7A^6 + 10A^5$. (CO 3)

Q 6.1 The circuit in the accompanying figure can be described by the differential equation

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 + R_2} & \frac{1}{R_2 C_1} \\ \frac{1}{R_2 C_2} & -\frac{1}{R_2 C_2} \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix},$$

where $y_1(t)$ and $y_2(t)$ are the voltages across the capacitors at time t . Suppose resistor R_1 is 1 ohm, R_2 is 2 ohms, capacitor C_1 is 1 farad, and C_2 is 0.5 farad, Find general formulas for $y_1(t)$ and $y_2(t)$ that describe how the voltages change over time. Further, if there is an initial charge of 5 volts on capacitor C_1 and 4 volts on capacitor C_2 , find the particular solution.



4.0
(CO 3,4,5)

Q 6.2 For the matrix A constructed in question 6.1, write a formula to compute A^5 using the concept of diagonalization. **No need to compute it.** 1.0
(CO 3,4)

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