

**JK Lakshmipat University, Jaipur**  
**INSTITUTE OF ENGINEERING AND TECHNOLOGY**  
End Term Examination (Regular/Backlog), November-December 2014  
B. Tech (All Branches), Semester I

Roll No.....

**MA 101: ENGINEERING MATHEMATICS - I**

**Time: 3 hours**

**Max. Marks: 100**

**Instructions to students:**

1. Do not write anything other than your roll number on question paper.
2. Use of calculator is not permitted.

**Q.1 Choose the correct answer for the following problems (10 questions @ 1 mark) 10**

- i. If  $u = x^4 + y^4 + 3x^2y^2$ , then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  will be;  
(a)  $4u$  (b)  $4$  (c)  $2u$  (d) none of these
- ii. Given a function  $u = f(x, y)$  and  $r = u_{xx}$ ,  $s = u_{xy}$ ,  $t = u_{yy}$ , then  $u$  is maxima if;  
(a)  $rt - s^2 = 0$  (b)  $rt - s^2 > 0, r < 0$  (c)  $rt - s^2 > 0, r > 0$  (d)  $rt - s^2 < 0$
- iii. An  $n^{\text{th}}$  degree curve has;  
(a) exactly  $n$  asymptotes (b) at most  $n$  asymptotes  
(c) at least  $n$  asymptotes (d) exactly  $n-1$  asymptotes
- iv. A curve  $f(x, y)$  is symmetric about  $x$ -axis if;  
(a)  $f(x, y) = f(y, x)$  (b)  $f(x, y) = f(-x, y)$   
(c)  $f(x, y) = f(x, -y)$  (d)  $f(x, y) = f(-x, -y)$
- v. If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  then divergence of  $\vec{r}$  is; (a) 0 (b) 1 (c) 2 (d) 3
- vi. Gradient of a surface  $S$  represents;  
(a) Scalar Field (b) Tangent Vector to  $S$   
(c) Normal vector to  $S$  (d) None of these
- vii. For changing cartesian to polar coordinate system in double integrals,  $dx dy$  is to be replaced by; (a)  $dr d\theta$  (b)  $r dr d\theta$  (c)  $r\theta dr d\theta$  (d)  $r^2 \sin \theta \cos \theta dr d\theta$
- viii.  $\iint_R dx dy$  represents;  
(a) Volume of the region  $R$  (b) Area of the region  $R$   
(c) Surface Area of region  $R$  (d) Surface Integral over region  $R$
- ix. For an even function  $f(x)$  over the interval  $-\pi < x < \pi$ , the Fourier coefficient  $a_0$  is; (a)  $\int_0^\pi f(x) dx$  (b)  $\int_0^{2\pi} f(x) dx$  (c) 0 (d)  $\pi$
- x. In a positive term series  $\sum u_n$ , if  $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \lambda$ , then the series is convergent if;  
(a)  $\lambda < 1$  (b)  $\lambda > 1$  (c)  $\lambda = 1$  (d) Not possible

**Q.2 Solve the following problems (4 questions @ 2.5 marks)**

**10**

- i. State Euler's theorem. Prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ , given  $u = \tan^{-1} \frac{x^3 + y^3}{x + y}$ .
- ii. Evaluate  $\int_0^{2a} \int_4^{x^2/4a} xy dx dy$ .
- iii. Find  $\vec{\nabla} \times \vec{F}$ , where  $\vec{F} = \text{grad} \left( \frac{x^3}{3} + \frac{y^3}{3} + \frac{x^2 y^2}{2} \right)$ .
- iv. Apply Cauchy's root test to test the convergence of the series  $\sum \frac{n^3}{3n}$ .



- Q.3A If  $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ , show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ . 05
- Q.3B Discuss the maxima and minima of  $f(x, y) = x^3 y^2 (1 - x - y)$ . 05
- Q.4A Find the asymptotes of the curve  $(x - 2y)^2 (x - y) - 4y(x - 2y) - (8x + 7y) = 0$ . 05
- Q.4B Trace the curve  $y^2 (2a - x) = x^3$ . 05
- Q.5A Trace the curve  $r^2 = a^2 \sin 2\theta$ . 05
- Q.5B Find the angle between normal to the surface  $xy = z^2$  at the points  $(4, 1, 2)$  and  $(3, 3, -3)$ . 05
- Q.6A Find the directional derivative of  $\phi = x^2 yz + 4z^2 x$  at the point  $P(1, -2, 1)$  in the direction of the vector  $2\hat{i} - \hat{j} - 2\hat{k}$ . 05
- Q.6B Prove that the vector field defined by  $\vec{V} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$  is irrotational. Find its scalar potential. 05
- Q.7A Evaluate the integral  $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  by changing to polar coordinates. 05
- Q.7B Evaluate the integral  $I = \int_0^1 \int_{ex}^e \frac{1}{\log y} dx dy$  by changing the order of integration. 05
- Q.8A Using line integral, find the total work done in moving a particle in a field of force given by  $\vec{F} = 3xy\hat{i} + 5z\hat{j} + 10x\hat{k}$  along the curve  $x = t^2 + 1, y = 2t^2, z = t^3$  from  $t = 1$  to  $2$ . 05
- Q.8B Use Green's theorem to evaluate  $\int_C [(y - \sin x)dx + \cos x dy]$ , where  $C$  is the triangle enclosed by the lines  $y = 0, x = \frac{\pi}{2}$  and  $y = \frac{2x}{\pi}$ . 05
- OR
- Q.8B The radius of a sphere is found to be 10 cm with a possible error of 0.02 cm. What is the relative error in computing the volume of sphere? 05
- Q.9A Evaluate  $I = \iint_S \vec{F} \cdot \hat{n} ds$  where  $\vec{F} = (x^2 + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$  and  $S$  is the surface of the plane  $2x + y + 2z = 6$  in the first octant. 05
- Q.9B Use Gauss's theorem to evaluate  $\iint_S \vec{F} \cdot \hat{n} ds$ , where  $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$  and  $S$  is the surface of a cube bounded by the planes:  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ . 05
- OR
- Q.9B A particle moves along the curve  $x = 3t^2, y = t^2 - 2t, z = t^3$ . Find its velocity and acceleration at  $t = 1$  in the direction of  $\hat{i} + \hat{j} - \hat{k}$ . 05
- Q.10 Obtain the Fourier series for  $f(x) = e^{-x}$  in the interval  $0 < x < 2\pi$ . 10
- OR
- Q.10 Find the volume of the solid generated by the revolution of the curve:  $y(a^2 + x^2) = a^3$  about its asymptote. 10

