

JK LAKSHMIPAT UNIVERSITY, JAIPUR**INSTITUTE OF ENGINEERING AND TECHNOLOGY**

Special End Term Examination (Regular/Backlog), January 2016

B. Tech.(All Branches), Semester III

Roll No.....

COURSE CODE: MA 301

Time: 3 hours

Max. Marks: 100

COURSE TITLE: Engineering Mathematics III

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Calculators are allowed.

Q.1A Objective type multiple choice questions (10 questions @ 1 mark) 10

i. What is the order of the pole at $z=1$ in $f(z) = \frac{1-e^z}{(z^2-1)^3}$

- (a) 1 (b) 2 (c) 3 (d) 0

The inverse Laplace transform of $\frac{3}{(s^2-9)}$

- (a) $\sin 3t$ (b) $\cos 3t$ (c) $\sinh 3t$ (d) $\cosh 3t$

ii. Which of the following is true for $f(z) = z^2$

- (a) Analytic everywhere (b) nowhere analytic
(c) Analytic only at Origin (d) Analytic everywhere except at Origin

iii. The value of $J_{-1/2}(x)$ is

- (a) $\sqrt{\frac{2}{\pi x}} \sin x$ (b) $\sqrt{\frac{2}{\pi x}} \cos x$ (c) $\sqrt{\frac{2}{\pi x}} \sin x$ (d) $\sqrt{\frac{2}{\pi x}} \cos x$

iv. The Laplace transform of $f(t) = te^t$ is

- (a) $\frac{1}{(s+1)^2}$ (b) $\frac{-1}{(s+1)^2}$ (c) $\frac{-1}{(s-1)^2}$ (d) $\frac{1}{(s-1)^2}$

v. $\int_0^\infty F_c(\alpha) \cos \alpha x d\alpha = \dots$, where $F_c(\alpha)$ denotes the Fourier cosine transform of $f(x)$

w.r.t. parameter α .

- (a) $f(x)$ (b) $\frac{\pi}{2(a^2+b^2)} f(x)$ (c) $\frac{\pi}{2} f(x)$ (d) none of these

vi. The inverse Laplace transform of $\frac{3}{(s^2-9)}$

- $\sin 3t$ (b) $\cos 3t$ (c) $\sinh 3t$ (d) $\cosh 3t$

vii	The residue of the function $f(z) = \frac{\cos z}{z^2}$ about its pole $z=0$, is (a) 1 (b) 2 (c) $4/3$ (d) zero	
viii	$\int_c \frac{1}{Z+2} dz$, where $c: z-1 =1$, is equal to (a) πi (b) $2\pi i$ (c) 0 (d) none of these	
ix	The value of $P_2(2)$ is (a) 5 (b) 5.5 (c) 6 (d) 6.5	
x	The Laplace transform of $\sqrt{t}$ is (a) not defined (b) $\sqrt{\frac{\pi}{s}}$ (c) $\frac{\pi}{\sqrt{s}}$ (d) $\left(\frac{1}{2}\right)! s^{-\frac{1}{2}}$	

Q.2	Short answer type questions (4 questions @ 2.5 marks)	10
(i)	Find the Value of $J_{3/2}(x)$ in terms of $\sin x$ and $\cos x$.	
(ii)	Explain convolution theorem for Laplace transform.	
(iii)	Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$.	
(iv)	Explain isoperimetric problems.	

Q.3A	Find the Laplace transform of the periodic function $f(t) = t^2, 0 < t < a$ and $f(t+a) = f(t)$	05
Q.3B	Prove that $u = x^2 - y^2 - y$ is harmonic. Find the harmonic conjugate of u and hence form the analytic function $f(z)$.	05

Q.4A	Find the Laplace transform of $f(t) = \begin{cases} 2 & , 0 < t < \pi \\ 0 & , \pi < t < 2\pi \\ \sin t & , t > 2\pi \end{cases}$.	05
Q.4B	Find the Fourier cosine transform of $f(x)$, where $f(x) = \begin{cases} e^{-ax}, & \text{for } 0 < x < a \\ 0, & \text{for } x > a \end{cases}$ Hence deduce the integrals $\int_0^\infty \frac{\cos \alpha x}{\alpha^2 + a^2} d\alpha$.	05

Q.5A	Expand the following in a series of Legendre polynomials $2x^3 + 2x^2 - x + 3$.	05
Q.5B	Find the inverse Laplace transform of $\frac{1}{s(s+a)^2}$.	05

Q.6A	$\frac{d}{dx} [J_n^2(x) + J_{n+1}^2(x)] = 2 \left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x) \right]$	05
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Q.6B	Find the Taylor's expansion of $f(z) = e^z \sin z$ about $z=0$.	05
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OR

Q.6B	Prove that the shortest distance between two given points in a plane is a straight line.	05
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Q.7A	Using Residue theorem, evaluate $\int_c \frac{z^2 + 4}{(z^3 + 2z^2 + 2z)} dz$, where c is the circle	05
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$$|Z - 1| = 5.$$

Q.7B	Evaluate the following integral using Cauchy integral formula	05
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$$\int_c \frac{e^z}{z(1-z)^3} dz, \text{ where } c \text{ is the circle } |Z - 1| = \frac{1}{2}.$$

OR

Q.7B	Solve the following initial value problem using Laplace transform	05
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$$y'' + 2y' + 5y = e^{-t} \cos t, \quad y(0) = 0, \quad y'(0) = 0$$

Q.8A	Using contour integration, evaluate the real integral $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos \theta} d\theta$.	10
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Q.9A	State and prove the orthogonality of Bessel's function.	10
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Q.9B	Solve the following system of simultaneous differential equations using Laplace transform	10
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$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t; \quad x(0) = 1, \quad y(0) = 0.$$

OR

Q.9B	Solve the following heat equation	10
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$$K \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad -\infty < x < \infty, \quad t > 0$$

$$\text{s.t. } u(x, 0) = f(x), \text{ where } f(x) = \begin{cases} u_0 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$