

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, April 2024
B. Tech. (CSE), Semester VI, 2023-24

Roll No.....

AS1207: NUMBER THEORY

Time: 3 hours

Max. Marks: 40

Instructions to students:

1. All questions are compulsory.
2. Do not write anything other than your roll number on the question paper.
3. Calculators are not allowed.

- Q.1** i) Find two positive integers, x such that $\phi(x) = 6$, where ϕ is Euler's phi-function. **3 x 1.0**
ii) State the number of quadratic residues and quadratic non-residues of 13. (CO-3,4,5)
iii) The bank identification number printed on a cheque consists of eight digits, $a_1 a_2 \dots a_8$, followed by a ninth check digit a_9 , where $a_9 \equiv 7a_1 + 3a_2 + 9a_3 + 7a_4 + 3a_5 + 9a_6 + 7a_7 + 3a_8 \pmod{10}$. What is the check digit following the eight-digit identification number 00185403?
- Q.2** Prove that if n is odd then $n^2 - 1$ is divisible by 8. **2.0**
(CO-1)
- Q.3** Show that if k is a positive integer, then $3k + 2$ and $5k + 3$ are relatively prime. **2.0**
(CO-1,2)
- Q.4** Check whether the linear congruence, $6x \equiv 15 \pmod{21}$, is solvable or not. If yes, find the solution. **2.0**
(CO-3)
- Q.5** Given an odd prime p and integers a and b that are not divisible by p , prove the following results for Legendre's symbol:
i) $(a/p)(b/p) = (ab/p)$
ii) $(a^2/p) = 1$ **2.0**
(CO-3,5)
- Q.6** Given that 3 is a primitive root of prime number, 43, find positive integers less than 43 having order 6 modulo 43. **2.0**
(CO-5)
- Q.7** Which of the integers, 8, 18, 35 and 49, will have a primitive root? Give proper reason for each integer. **2.0**
(CO-1,5)
- Q.8** If p is a prime and r is a primitive root of p then show that $r^{\frac{p-1}{2}} \equiv -1 \pmod{p}$. **2.0**
(CO-4,5)
- Q.9** For each positive integer n , show that
$$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$$
where μ is Möbius function. **2.0**
(CO-1,4)
- Q.10** Prove the Euclid's lemma which asserts that if $a \mid bc$, with $\gcd(a, b) = 1$, then $a \mid c$. **3.0**
(CO-1)
- Q.11** How many incongruent primitive roots does 14 have? Find a set of this many incongruent roots modulo 14. **3.0**
(CO-5)

- Q.12** Show that if n is a positive integer, then **3.0**
(CO-1,4)
- $$\phi(n) = \begin{cases} \phi(n), & \text{if } n \text{ is odd;} \\ 2\phi(n), & \text{if } n \text{ is even.} \end{cases}$$
- Q.13** For integer $k \geq 2$, show the following for $\sigma(n)$, which denotes sum of positive divisors of n : **4.0**
(CO-1,4)
- i) $n = 2^{k-1}$ satisfies the equation $\sigma(n) = 2n - 1$.
- ii) If $2^k - 1$ is prime, then $n = 2^{k-1}(2^k - 1)$ satisfies the equation $\sigma(n) = 2n$.
- Q.14** i) Using Fermat's theorem, show that **4.0**
(CO-1,3)
- $$1^p + 2^p + 3^p + \dots + (p-1)^p \equiv 0 \pmod{p}$$
- where p is an odd prime.
- ii) What is the last digit in the ordinary decimal representation of 3^{400} ?
- Q.15** Find the values of Legendre's symbol (p/q) in the four cases obtained from all combinations of $p = 13, 19$ and $q = 23, 41$. **4.0**
(CO-3)
