

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Backlog), May 2024
B. Tech. (All branches), Even Semester, 2024-25

Roll No.....

AS1406 : Partial Differential Equation and Fourier Transform

Time: 2 hours

Max. Marks: 30

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Mention all the assumptions for your answers clearly.

Q. 1.1 Classify the second-order linear PDE, $9u_{xx} = u_{tt} + 6u_t$ 2.0
(CO1)

Q. 1.2 Find the complex Fourier transform of $f(x) = \begin{cases} e^{-kx}, & x > 0 \text{ and } k > 0 \\ 0, & \text{otherwise} \end{cases}$ 2.0
(CO3)

Q. 1.3 Solve the PDE $z = e^{ax+by} \cdot f(ax - by)$ 2.0
(CO1)

Q. 1.4 Use Lagrange's method to solve the given first order PDE, 2.0
(CO1)
 $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

Q. 2.1 Find the Fourier sine and Fourier cosine transform of the following functions. 3+3
(CO3)
Also write the Fourier sine and cosine integrals for these functions.

(i) $f(t) = \begin{cases} k & \text{for } t < a \\ 0 & \text{for } t > a \end{cases}$

(ii) $f(t) = 2e^{-5t} + 5e^{-2t}$

Q. 2.2 Find the complete solution of the following nonlinear PDEs: 3+3
(CO1)
(i) $pq + p + q = 0$
(ii) $pxy + pq + qy = yz$

Q. 3.1 Determine the temperature distribution by solving 5.0
(CO2,3)
 $\frac{du}{dt} = \frac{\partial^2 u}{\partial x^2}$

$$\text{s.t. } u(0, t) = 0, \text{ for } t > 0, \quad \text{and } u(x, 0) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

$$u(x, 0) \text{ is bounded and } x > 0, t > 0$$

- Q. 3.2 Use the method of separation of variables to find the temperature $u(x,t)$ in a homogeneous bar of heat conducting material of length L cm with its ends kept at zero temperature and initial temperature given by $d x(L-X)/L^2$. 5.0
(CO1,2)

OR

- Q. 3.3 Write the Fourier cosine series expansions of the following function-
 $f(x) = \pi e^{-x} ; -\pi < x < 0$ 5.0
(CO2,3)
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all the best!