

JK Lakshmipat University, Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Special Backlog), January 2016
B. Tech/B. Tech + M. Tech (All Branches), Semester I

Roll No.....

MA 101: ENGINEERING MATHEMATICS - I

Time: 3 hours

Max. Marks: 100

Instructions to students:

1. Do not write anything other than your roll number on question paper.

Q.1 Choose the correct answer for the following problems (10 questions @ 1 mark) 10

- i. If $z = \cos\left(\frac{x}{y}\right) + \sin\left(\frac{x}{y}\right)$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ will be;
(a) z (b) $2z$ (c) 0 (d) none of these
- ii. The Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for the function $u = e^x \sin y$, $v = (x + \log \sin y)$ is;
(a) 1 (b) $\sin x \sin y - xy \cos x \cos y$ (c) 0 (d) $\frac{e^x}{x}$
- iii. The extreme values of the function $u = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$ are;
(a) $(0, 0)$ (b) $(2, 2)$ (c) (a) and (b) (d) none of these
(c) at least n asymptotes (d) exactly $n-1$ asymptotes
- iv. A curve $f(x, y)$ is symmetric about origin if;
(a) $f(x, y) = f(y, x)$ (b) $f(x, y) = f(-x, y)$
(c) $f(x, y) = f(x, -y)$ (d) $f(x, y) = f(-x, -y)$
- v. The maximum value of the directional derivative of the function $\phi = 2x^2 + 3y^2 + 5z^2$ at $(1, 1, -1)$ is; (a) 10 (b) -4 (c) $\sqrt{152}$ (d) 152
- vi. A force field $\vec{F}$ is called conservative if;
(a) $\vec{\nabla} \times \vec{F} = 0$ (b) $\vec{\nabla} \times \vec{F} \neq 0$ (c) $\vec{\nabla} \cdot \vec{F} = 0$ (d) $\vec{\nabla} \cdot \vec{F} \neq 0$
- vii. $\int_a^b 2\pi y^2 dx$ represents;
(a) Surface area of the solid of revolution of $x = f(y)$ about y-axis
(b) Surface area of the solid of revolution of $y = f(x)$ about y-axis
(c) Surface area of the solid of revolution of $x = f(y)$ about x-axis
(d) Surface area of the solid of revolution of $y = f(x)$ about x-axis
- viii. $\iint_R f(x, y) dx dy$ represents;
(a) Volume of the region bounded between $f(x, y)$ and R
(b) Area of the region bounded between $f(x, y)$ and R
(c) Surface area of the region bounded between $f(x, y)$ and R
(d) None of these
- ix. Given a surface ϕ , a vector $\frac{\vec{\nabla} \phi}{|\vec{\nabla} \phi|}$ represents;
(a) Unit normal vector to ϕ (b) Unit tangent vector to ϕ
(c) Unit normal vector to $\vec{\nabla} \phi$ (d) None of these
- x. Stoke's theorem connects;
(a) a line integral and a surface integral
(b) a surface integral and a volume integral
(c) a line integral and a volume integral (d) None of these

Q.2 Solve the following problems (4 questions @ 2.5 marks)

10

- i. Prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$, given $u = \ln \frac{x^4 + y^4}{x + y}$.
- ii. Evaluate $\int_{-1}^1 \int_2^4 y \, dx dy$.
- iii. Find the unit normal vector to the surface $f(x, y) = x^2 y^3 + xy$ at $(2, 1)$.
- iv. Find the divergence of the vector $\vec{V} = xy^2 \hat{i} + 2x^2 yz \hat{j} - 3yz^2 \hat{k}$ at the point $(1, 0, 2)$.

Q.3A If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 05

Q.3B Discuss the maxima and minima of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. 05

Q.4A Find the asymptotes of the curve $xy(x^2 - y^2) + x^2 + y^2 = a^2$. 05

Q.4B Trace the curve $y^2(a + x) = x^2(a - x)$. 05

Q.5A Trace the curve $r = 4(1 - \cos \theta)$. 05

Q.5B Find the angle between normal to the surface $xy = z^2$ at the points $(4, 1, 2)$ and $(3, 3, -3)$. 05

Q.6A Find the greatest rate of increase of $\phi = xyz + 4y^2x$ at the point $P(1, -2, 1)$. 05

Q.6B Prove that the vector field defined by $\vec{V} = (\sin y + z)\hat{i} + (x \cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational. Find its scalar potential. 05

Q.7A Evaluate the integral $I = \int_0^a \int_y^a \frac{x}{x^2 + y^2} dy dx$ by changing to polar coordinates. 05

Q.7B Evaluate the integral $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx dy$ by changing the order of integration. 05

Q.8A Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = x^2 \hat{i} + y^3 \hat{j}$ and the curve C is the arc of the parabola $y = x^2$ in the xy -plane from $(0, 0)$ to $(1, 1)$. 05

Q.8B Use Green's theorem to evaluate $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$, where C is the closed curve of the region bounded by $x = 0, y = 0$ and $x + y = 1$. 05

OR

Q.8B Find the percentage error in the area of an ellipse when an error of 1% is made in measuring its minor and major axis. 05

Q.9A Evaluate $I = \iint_S \vec{F} \cdot \hat{n} \, ds$ where $\vec{F} = (x^2 + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. 05

Q.9B Use Gauss's theorem to evaluate $\iint_S \vec{F} \cdot \hat{n} \, ds$, where $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ and S is the closed surface bounded by the planes: $z = 0, z = 1$, and the cylinder $x^2 + y^2 = 4$. 05

OR

Q.9B A particle moves along the curve $x = 2 \sin 3t, y = 2 \cos 3t, z = 8t$ at any time $t > 0$. Find the magnitude of velocity and acceleration at $t = 1$. 05

Q.10 Using Lagrange method of multipliers, divide 'a' into three parts so that the continued product of the first part, square of the second part and cube of the third part is maximum. 10

OR

Q.10 Find the volume of the solid generated by the revolution of the upper half of the circle $x^2 + y^2 = a^2$ about x -axis. 10