

JK Lakshmipat University, Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Backlog), November-December 2018

B. Tech (All Branches), Semester I

Roll No.....

MA102: Calculus and Linear Algebra

Time: 3 hours

Max. Marks: 100

Instructions to students:

1. Do not write anything other than your roll number on question paper.

Q.1 Choose the correct answer for the following problems (10 questions @ 1 mark) 10

- i. Given u as homogeneous equation of order 2, $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = ku$, then k is equal to;
(a) 3 (b) 1 (c) 2 (d) none of these
- ii. If an error of 1% is made in measuring its length and breadth, the % error in the volume of a parallelopied is (a) 0.2% (b) 0.02% (c) 3% (d) 0.03%
- iii. The rank of matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
(a) 1 (b) 2 (c) 4 (d) 8
- iv. $x = f(r, \theta), y = f(r, \theta)$ is called;
(a) Cartesian curve (b) Polar curve (c) Parametric curve (d) none of these
- v. A velocity vector is given as $\vec{V} = 5xy \hat{i} + 2y^2 \hat{j} + 3yz^2 \hat{k}$. The divergence of $\vec{V}$ at $(1, 1, 1)$ is; (a) 9 (b) 10 (c) 14 (d) 15
- vi. A curve $f(r, \theta)$ is symmetric about $\theta = 0^\circ$ if
(a) $f(r, \theta) = f(-r, \theta)$ (b) $f(r, \theta) = f(-r, -\theta)$
(c) $f(r, \theta) = f(r, -\theta)$ (d) $f(r, \theta) = f(r, \pi - \theta)$
- vii. The integral $\iint dx dy$ over a region R represents;
(a) Volume of region R (b) Area of region R
(c) Surface area of region R (d) None of these
- viii. The $r = a + b \cos \theta$ is called;
(a) Cardioid (b) Limacon (c) Equiangular Spiral (d) Laminscate of Bernoulli
- ix. In a surface integral $\iint_S \vec{F} \cdot \hat{n} ds$, $\hat{n}$ represents normal vector to;
(a) S (b) $\vec{F}$ (c) either (a) or (b) (d) None of these
- x. Given a surface $\emptyset$, a vector $\frac{\vec{\nabla} \emptyset}{|\vec{\nabla} \emptyset|}$ represents;
(a) Unit normal vector to $\emptyset$ (b) Unit tangent vector to $\emptyset$
(c) Unit normal vector to $\vec{\nabla} \emptyset$ (d) None of these

Q.2 Solve the following problems (4 questions @ 2.5 marks) 10

- i. Find f_{xyz} if $f(x, y, z) = 1 - 2xy^2z + x^2y$.
- ii. Explain the significance of gradient of a scalar field.
- iii. In which direction, magnitude of directional derivative of a function is maximum?
- iv. Under what conditions, a system of linear simultaneous equations is consistent?

- Q.3A State Euler's Theorem. If $u = \sin^{-1} \frac{x^2+y^2}{x+y}$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$. 05
- Q.3B Determine the area of $x^2 + y^2 = a^2$ bounded between positive x-axis and positive y-axis using integration. 05
- Q.4A If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 05
- Q.4B Trace the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$. 05
- Q.5A Write down step by step approach to trace a polar curve. 05
- Q.5B Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2x$ at the point P(1, 1, 1) in the direction of the vector $2\hat{i} - 2\hat{j} + \hat{k}$. 05
- Q.6A Explain the difference between divergence and curl of vector by taking a suitable example. Also highlight their physical significance. 05
- Q.6B If $\vec{A} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ then find a, b, c so that $\vec{A}$ is irrotational. 05
- Q.7A Calculate $\int_0^\pi \int_0^x xy \, dy dx$. 05
- Q.7B Evaluate the integral $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx dy$ by changing the order of integration. 05
- Q.8A A particle moves along the curve $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$, where t denotes time. Find the magnitude of acceleration along the tangent at time $t = 2$. 05
- Q.8B Find the Eigen values and Eigen vector of a matrix $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. 10
- OR
- Q.8B Find the volume of the solid generated by the revolution of the upper half of the circle $x^2 + y^2 = a^2$ about x-axis. 10
- Q.9A Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{bmatrix}$. 05
- Q.9B Use Green's theorem to evaluate $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$, where C is the closed curve of the region bounded by $x = 0, y = 0$ and $x + y = 1$. 10
- OR
- Q.9B Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Hence compute A^{-1} . 10