

Roll Number: _____

All questions are compulsory. There are 9 questions, each with sub-parts.
 Make appropriate assumptions wherever necessary.

1. Verify the following language.

(3+2+3 - COs 3,4)

- A. $L_1 = \{0^i 1^j 2^i 3^j \mid i, j \geq 1\}$ be a non Context-free language.
 B. $L_2 = \{ww^R \mid w \in \{0, 1\}^*\}$ be a Context-free language.
 C. $L_3 = \{a^n b^n c^n \mid n \geq 0\}$ be a recursively enumerable language.

2. Discuss the following statement.

(2x2- COs 4)

- A. K-tape Turing Machine $\equiv$ 1-tape Turing Machine
 B. The computation process of a Universal Turing Machine (U.T.M). Is the language accepted by a U.T.M. recursively enumerable?

3. Determine if the following post-correspondence problem (PCP) instance has a sol.

(1.5x2- COs 5)

A.

Index	A	B
1	bbab	a
2	ab	abbb
3	bba	aa
3	b	bbb

B.

Index	W	X
1	10	101
2	011	11
3	101	011

4. Justify whether the given statement holds.

(3+2 - COs 3,4)

- A. Every regular language is context-free. Is the converse of this statement true?

B. The class of C.F.L is closed under intersection.

5. Encode the given Turing machine in its binary form, based on the transition function

$$\delta(q_i, x_j) = (q_k, x_l, m),$$

where q_i, q_j are states, $0, 1, b$ are tape symbols, and $m \in \{L, R\}$ is the direction of the head.

(2 - COs 4)

State	0	1	b
q_1	$(q_2, 0, R)$	—	—
q_2	$(q_3, 0, R)$	—	—
q_3	$(q_4, 1, R)$	$(q_4, 1, L)$	$(q_4, 1, R)$
q_4	$(q_3, 1, L)$	$(q_5, 1, R)$	$(q_3, 1, L)$
q_5	—	—	—

6. Discuss the computation process of a Pushdown Automaton (PDA). Then, construct a PDA for the language.

$$L = \{a^i b^j c^k \mid i = j \text{ or } k \leq j \leq 2k\}.$$

(4 - COs 3)

7. Justify the equivalence between the class of languages accepted by Pushdown Automata (PDA) and the class of Context-Free Languages (CFLs). Illustrate your justification with a suitable example.

(3 - COs 3)

8. Construct a Deterministic Pushdown Automaton (DPDA) for the language

$$L = \{a^n b^m a^m b^n \mid n, m \geq 0\}$$

(2 - COs 3)

9. Answer the following questions:

- If L is a regular language, then L^R (the reversal of L) is also a regular language. Justify your answer.
- Let $L = \{a^l b^m c^n \mid l + m \leq n\}$. Justify that L is a nonregular language.
- Let $L = \{w \in \{0, 1\}^* \mid w \text{ begins and end with same symbol}\}$.
 - Is L a regular language?
 - Determine whether the strings "011" and "101" are accepted by L .

(3x3 - COs 1,2)