

JK Lakshmipat University, Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Regular), November 2016

B. Tech (All Branches), Semester I

Roll No.....

MA102: Calculus and Linear Algebra

Time: 3 hours

Max. Marks: 100

Instructions to students:

1. Do not write anything other than your roll number on question paper.

Q.1 Choose the correct answer for the following problems (10 questions @ 1 mark) 10

- i. The number of tangents at the Origin for the curve $x^2(x^2 + y^2) = a^2(x^2 - y^2)$ are ...; (a) 1 (b) 2 (c) 3 (d) none
- ii. If $f(x, y) = f(-x, -y)$ then the curve is symmetric about (a) y-axis (b) x-axis (c) origin (d) $y = x$
- iii. Divergence of $\vec{r}$ is; (a) 1 (b) 3 (c) 2 (d) 0
- iv. The rank of matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ (a) 1 (b) 2 (c) 3 (d) none of these
- v. The unit tangent vector of $\vec{r} = (1 + t^3)\hat{i} + te^{-t}\hat{j} + \sin 2t\hat{k}$ at $t = 0$ is (a) $\frac{\hat{i}+2\hat{k}}{\sqrt{5}}$ (b) $\frac{\hat{j}+2\hat{k}}{\sqrt{5}}$ (c) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{5}}$ (d) $\frac{\hat{i}+2\hat{j}}{\sqrt{5}}$
- vi. The triple integral $\iiint_R dx dy dz$ represents; (a) Volume of region R (b) Area of region R (c) Surface area of region R (d) None of these
- vii. If $f(x, y) = f(-x, y)$ then the curve is symmetric about (a) y-axis (b) x-axis (c) origin (d) $y = x$
- viii. If $u = e^{xyz}$, then u_z will be; (a) xyu (b) xzu (c) yzu (d) $xyzu$
- ix. If $0.X = B$ and $B \neq 0$, then X will have solution. (a) no (b) infinitely many (c) unique (d) can't say
- x. The Gauss divergence theorem connects; (a) a line integral and a surface integral (b) a surface integral and a volume integral (c) a line integral and a volume integral (d) None of these

Q.2 Solve the following problems (4 questions @ 2.5 marks) 10

- i. Check whether the vectors $\langle 1, 0, 1 \rangle$, $\langle 0, 1, 1 \rangle$ and $\langle 1, 1, 0 \rangle$ are linearly independent or dependent.
- ii. Evaluate $\int_0^{\pi/2} \int_0^{\cos \theta} r \sin \theta dr d\theta$.
- iii. If $xy^3 - yx^3 = 6$ is the equation of a curve, find the slope and the equation of the tangent line at the point (1, 2).
- iv. If $\vec{F} = y\hat{i} - x\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ from (0, 0) to (1, 1) along the parabola $y = x^2$.

Q.3A If $r^2 = x^2 + y^2$, prove that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r} \left\{ \left(\frac{\partial r}{\partial x} \right)^2 + \left(\frac{\partial r}{\partial y} \right)^2 \right\}$. 05

Q.3B Find $\frac{dy}{dx}$ when $y^{xy} = \sin x$. 05

Q.4A Calculate the percentage increase in the pressure p corresponding to a reduction of $\frac{1}{2}\%$ in the volume V , if the p and V are related by $pV^{1.4} = c$, where c is a constant. 05

Q.4B Trace the curve $y = (x^2 - x - 6)(x - 7)$. 05

Q.5A Trace the curve $r^2 = a^2 \sin 2\theta$. 05

Q.5B Find the values of a and b so that the surfaces $5x^2 - 2yz - 9z = 0$ may cut the surface $ax^2 + by^3 = 4$ orthogonally at $(1, -1, 2)$. 05

Q.6A State and prove Euler's theorem for $u = \sin^{-1} \left(\frac{x+2y}{\sqrt{x^6+y^6}} \right)$. 05

Q.6B Prove that $\vec{A} = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$ is irrotational. Find the scalar function f such that $\vec{A} = \nabla f$. 05

Q.7A Determine the area bounded by the curves $xy=2$, $4y=x^2$ and $y=4$. 05

Q.7B Evaluate the integral $\int_0^2 \int_1^{e^x} dy dx$ by changing the order of integration. 05

Q.8A Find the work done in moving a particle in a force field $\vec{F} = 3x^2\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$. 05

Q.8B The altitude of a right circular cone is 15 cm and is increasing at 0.2 cm/sec.. How fast is the volume changing. 05

OR

Q.8B Determine the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. 05

Q.9A Solve the following system of linear equations using any matrix method.
 $x+y+z=11$, $2x-6y-z=0$, $3x+4y+2z=0$. 05

Q.9B Use Green's theorem to evaluate $\int_C (xy + y^2)dx + x^2dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 05

OR

Q.9B Find the directional derivative of $\nabla \cdot (\nabla f)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$ where $f = 2x^3y^2z^4$. 05

Q.10 Verify Stoke's theorem for vector $\vec{A} = y^2\hat{i} + xy\hat{j} - xz\hat{k}$, where S is the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$. 10

OR

Q.10 Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 2 & -3 \\ 3 & 1 & -2 \end{bmatrix}$. Also verify Cayley 10

Hamilton theorem.