

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Regular/Backlog), April-May 2018

B. Tech.(All Branches), Semester II

Roll No.....

COURSE CODE: MA 202

Time: 3 hours

Max. Marks: 100

COURSE TITLE: Differential Equations and Complex Analysis

Instructions to students:

1. Do not write anything other than your roll number on question paper. Be neat and clean!!!!

Q.1A' Objective type multiple choice questions (10 questions @ 1 mark)

- i. An integrating factor transforms first-order differential equation to
 - (a) an exact form
 - (b) a homogeneous form
 - (c) a linear form
 - (d) None of them
- ii. In partial differential equation with three variables p, q, r are:
 - (a) $p = \frac{\partial u}{\partial x}, q = \frac{\partial p}{\partial y}, r = \frac{\partial q}{\partial z}$
 - (b) $p = \frac{\partial u}{\partial x}, q = \frac{\partial u}{\partial y}, r = \frac{\partial u}{\partial z}$
 - (c) $p = \frac{\partial u}{\partial x}, q = \frac{\partial v}{\partial y}, r = \frac{\partial f}{\partial z}$
 - (d) $p = \frac{\partial u}{\partial x}, q = \frac{\partial u}{\partial z}, r = \frac{\partial u}{\partial y}$
- iii. The general solution of an ODE contains a number of arbitrary constants which depend on the
 - (a) degree of the ODE
 - (b) linearity of the ODE
 - (c) order of the ODE
 - (d) homogeneity of ODE
- iv. The characteristics of a vibrating string can be treated as:
 - (a) Poisson's equation
 - (b) Laplace equation
 - (c) heat equation
 - (d) wave equation
- v. Cauchy Riemann equations do not ensure that the derivative of the function is independent of the path along which $\Delta z \rightarrow 0$.
 - (a) True
 - (b) False
 - (c) Partially true
 - (d) None of them
- vi. The solution of PDE: $xp + yq = x$ is
 - (a) $\phi(x^2/2 - y, y - z^2) = 0$
 - (b) $\phi(x - y, x - z) = 0$
 - (b) $\phi(x^2 + y^2, y - z) = 0$
 - (d) $\phi(x + y, x + z) = 0$
- vii. If f is analytic at a point, then
 - (a) f is an entire function
 - (b) f is harmonic function
 - (c) f is differentiable at that point
 - (d) None of them
- viii. The Laplace transform of a function $f(t)$ is $F(s)$. Then, $\lambda F(s)$ is
 - (a) $L(f(\lambda t))$
 - (b) $L(e^{\lambda t} f(t))$
 - (c) $L(f(t/\lambda))$
 - (d) $L(\lambda f(t))$
- ix. If $f(x) = e^x, -\pi < x < \pi$, then a_0 is
 - (a) $\frac{2}{\pi} \sin h\pi$
 - (b) $\frac{1}{\pi} \sin h\pi$
 - (c) $2 \sin h\pi$
 - (d) $\frac{4}{\pi} \sin h\pi$
- x. Laplace transforms provides a mathematical framework for the conversion of a function or derivative of a function from
 - (a) s domain to t domain in an algebraic form
 - (b) t domain to s domain in an integral form
 - (c) t domain to s in an algebraic form
 - (d) s domain to t domain in an integral form

Q.2	Short answer type questions (4 questions @ 2.5 marks)	10
Q.2A	Using Cauchy Riemann equations to show that $f(z) = z ^2$ is not analytical at any point.	
Q.2B	Find the Laplace transform of $\frac{1}{t}(\cos at - \cos bt)$.	
Q.2C	Determine (a) type, (b) order, (c) dependent variables and (d) independent variable of the PDE $\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$.	
Q.2D	Determine the poles of the following function and residue at each pole $f(z) = \frac{z^2}{(z-1)^2(z+2)}$.	
Q.3A	Evaluate the following integral using Cauchy integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where c is the circle $ z = \frac{3}{2}$.	05
Q.3B	Apply Laplace transform to solve the following ODE $\frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} - 8y = 0$.	05
Q.4A	Solve: $(1 + y^2) \frac{dx}{dy} + x = e^{\tan^{-1} y}$	05
Q.4B	Solve: PDE $\frac{p}{x^2} + \frac{q}{y^2} = z$.	05
Q.5A	Find Fourier sine transform of as $f(x) = \begin{cases} x, & 0 < x \leq 1 \\ 2-x, & 1 < x < 2 \\ 0, & x \geq 2 \end{cases}$	05
Q.5B	Solve: $(D^2 - 2D + 1)y = xe^x \sin x$	05
Q.6A	Show that $e^x(x \cos y - y \sin y)$ is harmonic function. Find the analytic function for which $e^x(x \cos y - y \sin y)$ is imaginary part.	05
Q.7A	In a transmission line, a dc signaling voltage is applied and the following relation is observed $\frac{d^2 v}{ds^2} - \alpha^2 v = 0$, where v is a voltage at a point on the line, s is the distance of the point from the end of the line, and α is the attenuation constant of the line. Find a relation for v and express it in hyperbolic form.	05
Q.7B	The free transversal vibration of thick rod is described by the partial differential equation $a^4 \frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0$. Prove that the solution of this partial differential equation by the method of separation of variables is of the following form: $u(x, t) = (c_1 \cos mx + c_2 \sin mx + c_3 \cosh mx + c_4 \sinh mx) \cos(\omega t + \epsilon)$, where $m^4 a^4 = \omega^2$ and $c_1, c_2, c_3, c_4, \omega$ and ϵ are constants.	10
OR		
Q.7B	Use Fourier transform to solve heat conduction equation which is given by: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, x > 0, t > 0$ subject to the conditions (i) $u=0$ when $x=0, t>0$, (ii) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$ when $t = 0$, (iii) $u(x, t)$ is bounded.	10
Q.8A	Write the statement of Residue theorem. Then, apply this theorem to solve $\int_C \frac{1}{z(z+1)^2(z+3)} dz$ where c is the circle $ z = 2$.	10
Q.9A	Solve the simultaneous equations:	10

$(D+2)x + 3y = 0$; $3x + (D+2)y = 2e^t$ given at $t=0$; $x=0$ and $y=0$.

- Q.9B A resistance R in series with inductance L is connected with $\text{emf} = E(t)$. The current ' i ' is given by $L \frac{di}{dt} + Ri = E(t)$. If the switch is connected at $t=0$ and disconnected at $t=a$, find the current ' i ' in terms of t . 10

OR

- Q.9B Explain even and odd functions. Then, discuss the Fourier series. Find the Fourier series for the function defined as $f(x) = \begin{cases} -1, & \pi \leq x < 0 \\ 0, & x = 0 \\ +1, & 0 < x \leq \pi \end{cases}$ 10