

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Regular/Backlog), Nov , 2016

B. Tech.(All Branches), Semester III

Roll No.....

COURSE CODE: MA 301

Time: 3 hours

Max. Marks: 100

COURSE TITLE: Engineering Mathematics III

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Calculators are allowed.

Q.1A	Objective type multiple choice questions (10 questions @ 1 mark)	10
i.	Function $f(z) = z^5 \cos\left(\frac{1}{z}\right)$ has type of singularity at $z=0$ (a) removable (b) essential (c) pole (d) zero	
ii.	The value of $\frac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$ is (a) 1 (b) 2 (c) 4/3 (d) 3/4	
iii.	Which of the following is true for $f(z) = z ^2$ (a) Analytic everywhere (b) nowhere analytic (c) Analytic only at Origin (d) Analytic everywhere except at Origin	
iv.	Which Fourier transform can be used to solve the following BVP $K' \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad -\infty < x < \infty, t > 0$ $s.t. u(x, 0) = g(x), \text{ where } g(x) = \begin{cases} p_0 & x \leq 4 \\ 0 & x > 4 \end{cases}$ (a) Fourier sine transform (b) Fourier cosine transform (c) complex Fourier transform (d) all of these	

v.	$J_4(x)$ is equal to (a) $-J_3(x)$ (b) $-J_4(x)$ (c) $J_4(x)$ (d) $J_2(x) - J_3(x)$
vi	The value of $L[(t+2)U(t-2); s] = \dots$ (a) $e^{-2s}\left(\frac{1}{s^2} + \frac{4}{s}\right)$ (b) $e^{2s}\left(\frac{1}{s^2} + \frac{4}{s}\right)$ (c) $e^{-2s}\left(\frac{1}{s^2} + \frac{2}{s}\right)$ (d) none of these .
vii	The infinite series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \infty$ is (a) convergent (b) divergent (c) oscillatory (d) unbounded
viii	Which of the following is true regarding poles of the function $f(z) = \frac{1-e^z}{(z^2-4)^3}$ (a) pole of order 3 at $z=2$ (b) pole of order 3 at $z=-2$ (c) simple pole at $z=0$ (d) both (a) and (b)
ix	The residue of the function $f(z) = \frac{\sin 2z}{z^2}$ about its pole $z=0$, is (a) 1 (b) 2 (c) $4/3$ (d) zero
x	The inverse Laplace transform of $\frac{1}{(2s-7)}$ (a) e^{-7t} (b) e^{7t} (c) $\frac{1}{2}e^{\frac{7t}{2}}$ (d) $\frac{1}{2}e^{\frac{-7t}{2}}$

Q.2	Short answer type questions (4 questions @ 2.5 marks)	10
(i)	Test the convergence of the series $\sqrt{\frac{1}{2^3}} + \sqrt{\frac{2}{3^3}} + \sqrt{\frac{3}{4^3}} + \sqrt{\frac{4}{5^3}} + \dots$	
(ii)	Define conformal mapping. Check whether the transformation $w=cz+b$ is a conformal mapping or not.	
(iii)	Find the Laplace transform of the periodic function	

	$f(t) = e^t$ for $0 < t < 2\pi$	
(iv)	Prove that $J_{-n}(x) = (-1)^n J_n(x)$ for $n=1, 2, 3, \dots$	
Q.3A	Find the Fourier series of $f(t) = \begin{cases} 0 & ; -2 \leq t \leq -1 \\ 1+t & ; -1 \leq t \leq 0 \\ 1-t & ; 0 \leq t \leq 1 \\ 0 & ; 1 \leq t \leq 2 \end{cases}$	05
Q.3B	Prove that $u = x^2 - y^2 - y$ is harmonic function. Find the harmonic conjugate of u and hence form the analytic function $f(z)$.	05
Q.4A	Verify Cauchy's integral theorem for $f(z)=z^2$ taken over the boundary of a square with vertices at $\pm 1 \pm i$ in counter-clockwise direction.	05
Q.4B	Evaluate $\int_c \frac{z^2 + 4}{(z^3 + 2z^2 + 2z)} dz$, where c is the circle $ Z = 3$.	05
Q.5A	Solve the following initial value problem using Laplace transform technique $(D^2 + 1)y = 2e^x$, at $x = 0, y = 0$ and $\frac{dy}{dx} = 2$.	05
Q.5B	Evaluate $\left[\int_0^\infty x e^{-x^8} dx \right] \times \left[\int_0^\infty x^2 e^{-x^4} dx \right]$.	05
Q.6A	Test the convergence of the series $1 + \frac{x}{1!}(\log a) + \frac{x^2}{2!}(\log a)^2 + \frac{x^3}{3!}(\log a)^3 + \dots \infty$	05
Q.6B	Find the inverse Laplace transform of $F(s) = \frac{16}{(s^2 + 4)^2}$ by convolution theorem.	05
	OR	
Q.6B	Prove the orthogonality of Bessel's function.	05
Q.7A	Find the Fourier cosine transform of $f(x) = \begin{cases} \sin x & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } x > \pi \end{cases}$	05

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Q.7B	Establish the following $\frac{d}{dx} [J_n^2(x) + J_{n+1}^2(x)] = 2 \left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x) \right]$	05
	OR	
Q.7B	Find the value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\sin 8\theta}}{\sqrt{\cos \theta}} d\theta$.	05
Q.8A	Obtain the Fourier sine and cosine series of $f(x) = \sin x$ in the interval $0 < x < \pi$.	10
Q.9A	Using contour integration, evaluate the real integral $\int_0^{2\pi} \frac{d\theta}{(5 - 3 \sin \theta)^2}$.	10
Q.9B	Find the temperature $u(x, t)$ in a semi-infinite bar with $\frac{\partial u}{\partial x} = \mu$ when $x=0$ and with initial temperature 0. Assume that $\frac{\partial u}{\partial x} \rightarrow 0$ as $x \rightarrow \infty$ and is bounded.	10
	OR	
Q.9B	Solve the following system of simultaneous differential equations using Laplace transform $2 \frac{dx}{dt} + \frac{dy}{dt} - x - y = e^{-t}, \quad \frac{dx}{dt} + \frac{dy}{dt} + 2x + y = e^t; \quad x(0)=2, y(0)=1.$	10