

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Make Up), December 2024

B. Tech. (CSE), Semester V, 2024-25

Roll No.....

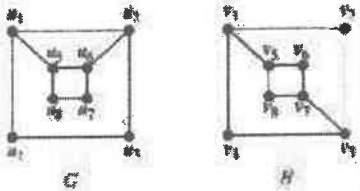
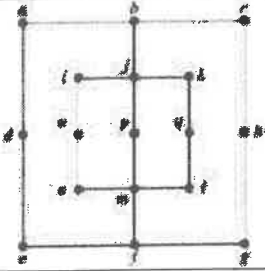
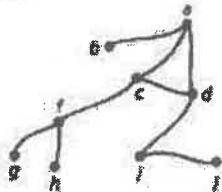
CS1142: Discrete Mathematics

Time: 3 hours

Max. Marks: 40

Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. Non programmable calculators are allowed.
3. Mention all the assumptions for your answers clearly.

Q. 1	a) Disprove the statement: There exist an integer $m \geq 3$ such that $m^2 - 1$ is prime. b) Determine the exponential generating function for the sequence of factorials: $0!, 1!, 2!, \dots, n!, \dots$ c) Consider the cyclic group, $G = (\mathbb{Z}_6, +)$ and $H = \{0, 2, 4\}$ be a nonempty subset of G. I. Prove or disprove: H is a subgroup of G. II. Find all generators of the cyclic group, G.	1+1+2 (CO2,4,5)
Q. 2	How many ways are there to place 10 indistinguishable balls into eight distinguishable bins? Write the solution with complete explanation.	2 (CO5)
Q. 3	Determine the number of vertices, edges and the number of regions for following planar graph. Show that the answer satisfy the Euler's Theorem for connected planar graphs. 	2 (CO5)
Q. 4	Determine whether the given pair of graphs are isomorphic or not. Give proper justification for your answer. 	2 (CO2,5)
Q. 5	Does the following graph has a Hamilton path? If so, find such a path. If it does not, give an argument to show why no such path exists. 	2 (CO5)
Q. 6	How many spanning subgraphs and connected spanning subgraphs are there for the following graph G. Draw all. 	2 (CO5)

Q. 7	Verify the given logical equivalence using truth table, where p, q, r are propositional variables, $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$	2 (CO1)
Q. 8	Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$.	2 (CO1)
Q. 9	The number of bacteria in a culture is 1000 (approx.). And this number increases 205% every two hours. Write a recurrence relation to formulate this problem and determine the number of bacteria present after one day (24 hours).	2 (CO5)
Q. 10	Let $(Z \times Z, \oplus)$ be an abelian group where $(a, b) \oplus (c, d) = (a + c, b + d)$, here $a + c$ and $b + d$ are computed using ordinary addition in Z and let $(Z, +)$ be an additive group. Let $f : Z \times Z \rightarrow Z$ be the function defined by $f(x, y) = x - y$. i. Prove that f is a homomorphism onto Z . ii. Determine all $(a, b) \in Z \times Z$ with $f(a, b) = 0$. iii. Find $f^{-1}(7)$.	2+1+1 (CO2,4)
Q. 11	Suppose Friendly Airways has nine daily flights as follows: 103 Atlanta to Houston 203 Boston to Denver 305 Chicago to Miami 106 Houston to Atlanta 204 Denver to Boston 308 Miami to Boston 201 Boston to Chicago 301 Denver to Reno 401 Reno to Chicago Describe the data by means of a labeled directed graph G . Can a passenger take a tour of all 9 destinations without repeating any destination? If yes write all such paths (tours). If not Justify your answer.	3 (CO5)
Q12.	A machine that inserts letters into envelopes goes haywire and inserts letters randomly into envelopes. What is the probability that in a group of 100 letters i. no letter is put into the correct envelope? ii. exactly one letter is put into the correct envelope? iii. exactly 99 letters are put into the correct envelopes? iv. all letters are put into the correct envelopes?	4 (CO2,5)
Q. 13	What is the number of integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ that satisfy $1 \leq x_1 \leq 5, -2 \leq x_2 \leq 4, 0 \leq x_3 \leq 5, 3 \leq x_4 \leq 9$.	4 (CO5)
Q. 14	Let U be the power set of set of natural numbers and P is the power set of set $A = \{0, 1, 2, 3\}$ and R be a relation "is a super set of" defined on P . Such that (U, R) is a poset. i. Check whether (P, R) is a poset or not. Justify properly. ii. Draw the Hasse diagram of (P, R) . iii. Check whether (P, R) is a lattices or not. Justify. iv. Write a lower and an upper bound of P . v. Identify the least and greatest elements of (P, R) .	5 (CO 3)

All the best!