

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Regular/Backlog), November 2017

B. Tech (All Branches), Semester I

Roll No.....

COURSE CODE: MA 102

Time: 3 hours

Max. Marks: 100

COURSE NAME: Calculus and Linear Algebra

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Assume suitable data wherever required and mention it clearly.

Q.1A	Objective type multiple choice questions (10 questions @ 1 mark)	10
i.	Rank of the matrix $A = \begin{bmatrix} 3 & 2 & -9 \\ -6 & -4 & 18 \\ 12 & 8 & -36 \end{bmatrix}$ is (a) 1 (b) 2 (c) 3 (d) None of these	
ii.	The line integral of function $\vec{F} = yz\hat{i}$, in counterclockwise direction, along the circle $x^2+y^2 = 1$ at $z=1$ is (a) π (b) $-\pi$ (c) 2π (d) None of these	
iii.	If vectors A and B are irrotational, then (a) $A \times B$ is irrotational (b) $A \cdot B$ is solenoidal (c) $A \times B$ is solenoidal (d) None of these	
iv.	If $u=x+y$ and $v=xy$ then the value of $J\left(\frac{u,v}{x,y}\right)$ is (a) xy (b) $x+y$ (c) x^2-y (d) $x-y$	
v.	Let A be a 3×3 matrix with rank 2; then $AX=0$ has (a) it has two independent solutions (b) it has one independent solution (c) only the trivial solution $X=0$ (d) it has three independent solutions	
vi.	What is the area of the region enclosed by $y^2=x$ and $y=x-2$? (a) $11/2$ (b) $9/2$ (c) $5/2$ (d) $1/2$	
vii.	The divergence of the vector field $x^2z\hat{i} + xy\hat{j} - yz^2\hat{k}$ at $(1,-1,1)$ is (a) 0 (b) 3 (c) 5 (d) 6	
viii.	Total derivative of the function 'xy' (a) $dx \cdot dy$ (b) $dx+dy$ (c) $x dx+y dy$ (d) $x dy+y dx$	
ix.	The value of $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$ is (a) 2 (b) π (c) $\pi/2$ (d) None of these	
x.	The time 'T' of a complete oscillation of a simple pendulum of length 'L' is governed by the equation $T = 2\pi \sqrt{\frac{L}{g}}$ where g is a constant. By what percentage should the length be changed in order to correct a loss of 2 minutes per day? (a) -0.118% (b) -0.568% (c) -0.378% (d) -0.278%	
Q.2	Short answer type questions (4 questions @ 2.5 marks)	10

Q.2A	Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point $P(2,1,3)$ in the direction of vector $\vec{a} = \hat{i} - 2\hat{k}$.	
Q.2B	For $x = u, y = u \tan v, z = w$, verify $\frac{\partial(x,y,z)}{\partial(u,v,w)} \cdot \frac{\partial(u,v,w)}{\partial(x,y,z)} = 1$.	
Q.2C	The acceleration of a particle, which starts from rests, is $6t\hat{i} + 8t^3\hat{j} - 12t^2\hat{k}$, at any instant $t \geq 0$. Find its velocity at time 't'.	
Q.2D	Write the statement of Gauss divergence theorem and give one application where we can apply result of divergence theorem.	
Q.3A	Show that the given system of equations : $x + y + z = 6, x + 2y + 3z = 14, x + 4y + 7z = 30$ are consistent then solve them.	05
Q.3B	If $x^x y^y z^z = c$, show that at $x = y = z, \frac{\partial^2 z}{\partial x \partial y} = -(x \log x)^{-1}$.	05
Q.4A	Find the volume of the solid generated by revolving the region bounded by the curves $y = 3 - x^2$ and $y = -1$ about the line $y = -1$.	05
Q.4B	Show that the curvature at the point $(3a/2, 3a/2)$ on the folium $x^3 + y^3 = 3axy$ is $-8\sqrt{2}/3a$.	05
Q.5A	Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	05
Q.5B	Evaluate $\iint_S \vec{F} \cdot \hat{n} dS$ where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.	05
Q.6A	State Euler's theorem then verify $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ if $u = \log_e \left(\frac{x^4 + y^4}{x + y} \right)$.	05
Q.7A	By changing the order of integration, evaluate $\int_0^a \int_{y^2/a}^y \frac{y}{(a-x)\sqrt{(ax-y^2)}} dx dy$.	05
Q.7B	Find the arc length of the curve $x = \log y$ between the ordinates $4/5$ and $5/4$. Also, determine the unit normal vector for $r(t) = t\hat{i} + t^2\hat{j}$ when $t=1$.	10
OR		
Q.7B	Prove that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find a scalar function $\phi(x,y,z)$. "Does the value of a line integral $\int_C \vec{F} \cdot d\vec{r}$ change if we integrate from the same initial point to the same terminal point but along another path". Justify your answer.	10
Q.8A	Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.	10
Q.9A	Write all properties for vector spaces. If $V_3(R)$ is a vector space then show that $S = \{(0,1,-1), (1,1,0), (1,0,2)\}$ is a basis of V_3 and hence find co-ordinates of vector $(1,0,-1)$ with respect to this basis.	10

Q.9B

10

Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence, find A^{-1} . Also, discuss the rank of the matrix.

OR

Q.9B

10

It is required to build a parabolic satellite dish, whose shape will be formed by rotating a parabolic curve $y = ax^2$ about the y-axis. If the dish is to be having a diameter of 10 ft and a maximum depth of 2 ft, find the value of 'a' and the surface area of the dish.

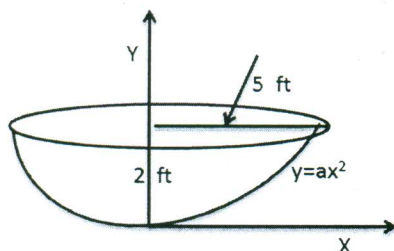


Fig. Parabolic satellite dish.