

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY
End Term Examination (Regular/Backlog), November-December 2015
B. Tech. / B. Tech. + M. Tech (CSE & ECE), Semester V

Roll No.....

MA 502: OPTIMIZATION TECHNIQUES

Time: 3 hours

Max. Marks: 100

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Draw the graphs neatly and legible. Write your roll number and question number on the graph paper.

Q.1 Choose the correct answer for the following problems (10 questions @ 1 mark) 10

- i. In which of the following methods, it is not necessary to calculate all the entries in the simplex table for each iteration;
(a) Dual simplex method (b) Revised simplex method
(c) Two phase method (d) None of these
- ii. If the primal variable x_j is unrestricted in sign, then the corresponding dual constraint will hold with the;
(a) ' $\leq$ ' sign (b) ' $\geq$ ' sign (c) '=' sign (d) None of these
- iii. In a balanced transportation problem with m sources and n destinations, the number of basic variables in the allotment is;
(a) $m + n - 1$ (b) $m + n$ (c) $m + n + 1$ (d) $m.n$
- iv. The assignment problem is a special case of;
(a) non-linear programming problem (b) a transportation problem
(c) a quadratic programming problem (d) none of these
- v. Graphical method is used generally for which of the following sequencing problems?
(a) n jobs 2 machines (b) n jobs k machines
(c) 2 jobs k machines (d) both (b) and (c)
- vi. Any change in costs and availabilities of an LPP simultaneously may disturb;
(a) only optimality (b) only feasibility
(c) Both optimality and feasibility (d) Neither optimality nor feasibility
- vii. The utilization factor for a system represents;
(a) The probability that there is no one in the system
(b) The probability that the service facility is being used
(c) The average number of customers in the queue
(d) The steady state average waiting time
- viii. Which of the following are not assumptions of the basic EOQ model?
(a) Demand is known and constant (b) Inventory receipt is instantaneous
(c) Quantity discounts are not possible (d) None of these
- ix. Given an activity's optimistic, most likely, and pessimistic time estimates of 6, 10, 14 days, respectively, the PERT time for this activity is;
(a) 5 (b) 10 (c) 7 (d) 12
- x. The first step in simulation is to;
(a) set up possible courses of action for testing (b) construct a numerical model
(c) validate the model (d) define the problem

Q.2 Solve the following problems (4 questions @ 2.5 marks)

10

i. Write down the dual of the following primal LP problem:

$$\text{Maximize } Z = 2x_1 + 3x_2 + x_3$$

$$\text{Subject to: } 4x_1 + 3x_2 + x_3 = 6; x_1 + 2x_2 + 5x_3 = 4 \text{ and } x_1, x_2, x_3 \geq 0$$

ii. Find the range for the values of u and v , which will render the entry in the position (2, 2) i.e., '7' as a saddle point for the following game.

	B ₁	B ₂	B ₃
A ₁	2	4	5
A ₂	8	7	u
A ₃	5	v	9

iii. Highlight the difference between deterministic and stochastic operations research (OR) models. Also explain the problem solving process in an OR model.

iv. Explain the transportation model for milk collection and distribution process at dairy plants.

Q.3A A manufacturer wants to ship 8 load units of his products as shown in the following table.

05

	D ₁	D ₂	D ₃	Available
O ₁	50	30	220	1
O ₂	90	45	170	3
O ₃	250	200	50	4
Required	4	2	2	

The matrix gives the mileage from origin O_i to D_j . The shipping costs are Rs. 10 per load unit per mile. Find out the shipping route that manufacturer should opt incurring minimum shipping cost? What will be the optimal cost?

Q.3B Solve the following assignment problem.

05

	1	2	3	4
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

Q.4A A shopkeeper maintains a stock of Britannia bread. On the basis of the shopkeeper's previous experience, the daily demand for the bread along with associated probabilities is given in the following table.

05

Daily demand (number)	0	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

Estimate the daily average demand for the bread by simulating the demand for 10 days. Use the random numbers: 25, 39, 65, 76, 12, 05, 73, 89, 19, and 49.

Q.4B State the Kuhn-Tucker conditions. Write down the Kuhn-Tucker conditions to get an optimal solution of the following Non-linear programming problem:

05

$$\text{Maximize } Z = x^2 + y^2 + z^2 + 20x + 10y$$

$$\text{Subject to: } x \geq 40, x + y \geq 80, x + y + z \geq 120 \text{ and } x, y, z \geq 0.$$

- Q.5A A vessel that can carry a maximum load of seven units is loaded with three items. The weight per unit of different item and their values are given in the following table. 05

Item	Weight	Value
1	1	20
2	3	90
3	2	70

Use dynamic programming to find the number of units of each item that should be loaded in the vessel to maximize the value of the loaded item.

- Q.5B Use graphical method to find the optimal strategy of each player and the value of the following game. 05

		Player B			
		1	2	3	4
Player A	1	2	3	11	8
	2	7	5	2	7
	3	6	4	-4	-9

- Q.6A A tollgate is operated on a highway where vehicles arrive according to Poisson distribution with a mean frequency of 1.2 vehicles per minute. The time of completing toll tax payment is an exponential distribution with mean 20 seconds. Find (a) P_0 (b) L_s (c) L_q (d) W_s (e) W_q . 05

- Q.6B Six jobs are to be performed on three machines M_1 , M_2 and M_3 , in the order $M_1M_2M_3$. The processing times are given as per the following table. 05

Jobs → Machines ↓	J_1	J_2	J_3	J_4	J_5	J_6
M_1	12	10	9	14	7	9
M_2	7	6	6	5	4	4
M_3	6	5	6	4	2	4

Determine the optimal sequence and the total elapsed time associated with it.

- Q.7A A commodity is to be supplied at a constant rate of 200 units per day. Supplies of any amount can be held at any required time, but each order costs Rs. 50. The cost of holding the commodity in the inventory is Rs. 2.00 per unit per day, whereas the delay in the supply of the item induces a penalty of Rs. 10.00 per unit per delay of one day. Find the optimal policy (q, t) , where t is the reorder cycle period and q is the inventory level after the reorder. What would the best policy be if there is no scope of any delay in supply? 05

- Q.7B Use the graphical method to minimize the total time elapsed to complete jobs 1 and 2 based on the following details. 05

Job 1 Machines					
Sequence	A	B	C	D	E
Time (h)	3	4	2	6	2

Job 2 Machines					
Sequence	B	C	A	D	E
Time (h)	5	4	3	2	6

- Q.8 A small project is composed of nine activities whose time estimates are listed in the table below: 10

Activity	1-2	2-3	2-4	2-5	3-7	4-6	5-6	6-7	7-8
Duration (Optimistic)	2	8	4	4	3	7	3	2	2
Duration (Most likely)	5	11	7	9	5	10	7	3	4
Duration (Pessimistic)	8	20	16	20	13	13	17	10	6

- Draw the project network diagram.
- Find the expected duration and variance of each activity.
- Find the critical path and expected project length.
- Calculate the variance and standard deviation of project length. What is the probability that the project will be completed in 40 days?
- What is the time duration that will provide 95% probability of its completion time?

- Q.9* Solve the following LPP using the revised simplex method. 10

$$\text{Maximize } Z = 2x_1 - x_2 + x_3$$

Subject to: $3x_1 + x_2 + x_3 \leq 60$; $x_1 - x_2 + 2x_3 \leq 10$; $x_1 + x_2 - x_3 \leq 20$ and $x_1, x_2, x_3 \geq 0$.

OR

- Q.9 A company wishes to determine an investment strategy for each of the next four years. Five investment types have been selected, investment capital has been allocated for each of the coming four years, and maximum investment levels have been established for each investment type. An assumption is that amounts invested in any year will remain invested until the end of the planning horizon of four years. The following table summarizes the data for this problem. The value in the body of the table represents net return on investment of one rupee upto the end of the planning horizon. For example, a rupee invested in investment type B at the beginning of year 1 will grow to Rs. 1.90 by the end of the fourth year, yielding a net return of Re. 0.90. 10

Investment made at the beginning of year	Investment type					Rupees available (in 000's)
	A	B	C	D	E	
	Net return data					
1	0.80	0.90	0.60	0.75	1.00	500
2	0.55	0.65	0.40	0.60	0.50	600
3	0.30	0.25	0.30	0.50	0.20	750
4	0.15	0.12	0.25	0.35	0.10	800
Maximum Rupees investment (in 000's)	750	600	500	800	1000	

The objective in this problem is to determine the amount to be invested at the beginning of each year in an investment type so as to maximize the net rupee return for the four year period. Can this problem be solved as Transportation problem? If yes, find an optimal solution and calculate the net return on investment for the planning horizon of four years period.

Q.10 Solve the following LPP using the dual simplex method. 10

$$\text{Minimize } Z = 2x_1 + 3x_2 + 5x_3$$

Subject to: $x_1 + 2x_2 + 3x_3 \geq 2$; $2x_1 - x_2 + x_3 \geq 3$ and $x_1, x_2, x_3 \geq 0$.

OR

Q.10 A repairman is to be hired to repair machines which breakdown at an average rate of Rs. 5.00 per hour. The breakdown follows Poisson distribution. The productive time of a machine is considered to cost Rs. 20 per hour. Two repairmen have been interviewed – one is slow & cheap while the other is fast but expensive. The slow repairman charges Rs. 10 per hour & he services breakdown machines at the rate of 6.00 per hour. The fast repairman demands Rs. 40 per hour & he repairs at an average rate of 10 per hour. Which repairman should be hired? (Assume 8 hours a day). 10