

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Special Backlog), November 2016
B. Tech in Computer Science Engineering, Semester IV

Subject Code: CSE402

Roll No.....

Subject Name: Discrete Structures

Time: 3 hours

Max. Marks: 100

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Assume suitable data wherever required and mention it clearly.
3. Writing appropriate units, nomenclature, and drawing neat sketches/schematics/ flow chart wherever, required are an integral part of the answer.

Q.1A Objective type multiple choice questions (10 questions @ 1 mark) 10

- i. Which of the following statement is the negation of the statement, "3 is odd or -5 is negative"?
(a) 3 is even and -5 is not negative (b) 3 is odd and -5 is not negative
(c) 3 is even or -5 is not negative (d) 3 is odd or -5 is not negative
- ii. $p \rightarrow q$ is logically equivalent to
(a) $\neg q \rightarrow p$ (b) $\neg p \rightarrow q$ (c) $\neg p \wedge q$ (d) $\neg p \vee q$
- iii. Let $Q(x, y)$ denote " $x - y = 0$ ". The truth values of the quantifications $\forall x \exists y Q(x, y)$ and $\exists y \forall x Q(x, y)$ are respectively
(a) true, true (b) true, false (c) false, true (d) false, false
- iv. Tautology $[(p \vee q) \wedge \neg p] \rightarrow q$ represents ----- rule of inference.
(a) Modus tollens (b) Disjunctive syllogism (c) Simplification (d) Resolution
- v. If e denotes the number of total edges in a graph, then the sum of degrees of all the vertices in a graph is
(a) e (b) $e/2$ (c) e^2 (d) $2e$
- vi. If G is a connected planar simple graph with e edges and v vertices, where $v \geq 3$, then
(a) $3v - 6 \geq e$ (b) $3v - 6 \leq e$ (c) $3e - 6 \geq v$ (d) $3e - 6 \leq v$
- vii. If $f(x) = \cos x$ and $g(x) = 2x^2 - 1$, then $(g \circ f)(x)$ will be invertible for the domain
(a) $[0, \pi]$ (b) $[0, \frac{\pi}{2}]$ (c) $[\frac{-\pi}{2}, 0]$ (d) $[\frac{-\pi}{4}, \frac{\pi}{4}]$
- viii. If $f(x) = \sin x$ and $g(x) = 1 - 2x^2$, then $(g \circ f)(x)$ will be invertible for the domain
(a) $[0, \pi]$ (b) $[0, \frac{\pi}{2}]$ (c) $[\frac{-\pi}{2}, 0]$ (d) $[\frac{-\pi}{4}, \frac{\pi}{4}]$
- ix. Let $R = \{(a, b) \mid a \text{ "divides" } b\}$ be a relation on the set of natural numbers. The relation is
(a) reflexive and transitive (b) reflexive only
(c) an equivalence relation (d) partial order relation
- x. Let $A = \mathbb{Z}$ and let R is a relation defined on A as follows:
 $R = \{(a, b) \mid a \text{ and } b \text{ yield the same remainder when divided by } 2\}$. The relation is
(a) reflexive and transitive (b) reflexive only
(c) an equivalence relation (d) partial order relation

Q.2 Short answer type questions (4 questions @ 2.5 marks) 10

- Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.
- Is it possible to draw a simple graph with 4 vertices and 7 edges? Justify.
- Show that if 7 colors are used to paint 50 bicycles, at least 8 bicycles will be painted with same color.
- Let $A = \{1, 2, 3\}$ and let R and S be relations on A whose matrices are given below. Compute $M_{S \circ R}$, where $S \circ R$ denotes the composition of R and S .

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Q.3A Show that $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology. 05

Q.3B Use rules of inferences to show that the hypotheses "Randy works hard", "If Randy works hard, then he is a dull boy", and "If Randy is dull boy, then he will not get the job" imply the conclusion "Randy will not get the job". 05

Q.4A Prove that if n is a positive integer, then n is odd if and only if $5n+6$ is odd. 05

Q.4B Encrypt the message "LAST IN FIRST OUT" by translating the letters into numbers, applying the encryption function $f(p) = (p+13) \bmod 26$, and then translating the numbers back into letters. 05

Q.5A Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer. 05

Q.5B Find GCD (2000, 1250) and LCM (2000, 1250) using the prime factorization of integers and verify that $\text{GCD}(2000, 1250) * \text{LCM}(2000, 1250) = 2000 * 1250$. 05

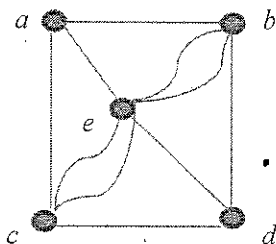
Q.6A Verify that the program segment 05

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x := 2
z := x + y
if y > 0 then
  z := z + 1
else
  z := 0
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is correct with respect to the initial assertion $y = 3$ and the final assertion $z = 6$.

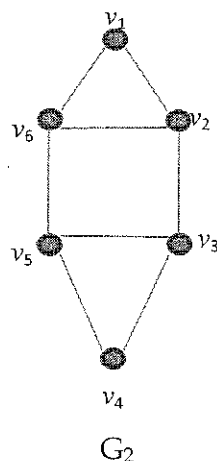
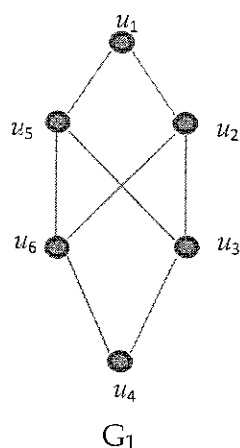
Q.6B i. Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane? 2.5

ii. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exist. 2.5

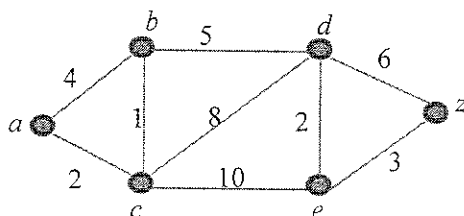


OR

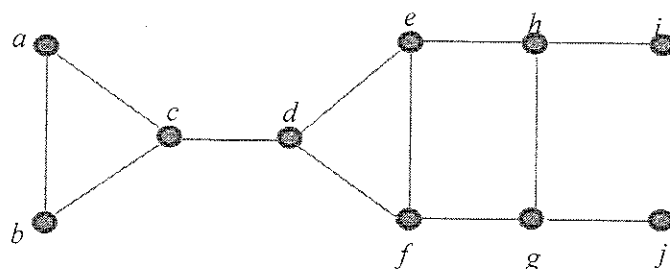
Q.6B Determine whether the graphs G_1 and G_2 shown in figure below are isomorphic. 05



- Q.7A Use Kruskal's algorithm to find a minimum spanning tree in the weighted graph given below: 05



- Q.7B Use breadth-first search to produce a spanning tree the simple graph shown below. Choose a as the root of spanning tree. 05



OR

- Q.7B Suppose that 100 of the 120 Computer Science students at a college choose at least one of the programming languages C, Java and PHP. Also suppose that 65 study C, 45 study Java, 42 study PHP, 20 study C and Java, 25 study C and PHP and 15 study Java and PHP. 05
- Find the number of students who study all three languages.
 - Fill in the correct number of students in each of the eight regions of Venn diagram.
 - Determine the number of students who study exactly two languages.

- Q.8A i. If there are 12 elements in A_1 and 18 elements in A_2 . Determine the number of elements in $A_1 \cup A_2$ for each of the following cases: 03
- $A_1 \cap A_2 = \phi$
 - $|A_1 \cap A_2| = 1$
 - $A_1 \subseteq A_2$
- ii. Verify the statement: $(A \cap B) \cup \overline{A} \cap \overline{B} \cap C = U$ 02
- iii. Consider the function $f(x) = x^2$ from the set of all nonnegative real numbers to 05

the set of all nonnegative numbers. Is f invertible? If it is, what is its inverse?

- Q.9A i. Let $A = B = C = \mathbb{R}$ and Let $f: A \rightarrow B$, $g: B \rightarrow C$ be defined by $f(x) = x + 1$ and $g(x) = x^2 + 2$. Compute the following: 05
- (a) $(f \circ g)(2)$ (b) $(g \circ f)(-2)$ (c) $(f \circ f)(2)$ (d) $(g \circ g)(-1)$
- ii. Let $R_1 = \{(1, 2), (2, 3), (3, 4)\}$ and $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ be relations from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Compute the following: 05
- (a) $R_1 \cup R_2$ (b) $R_1 \cap R_2$ (c) $R_1 - R_2$ (d) $R_2 - R_1$ (e) R_1^{-1}
- Q.9B i. Determine the transitive closure R^+ and reflexive-transitive closure R^* for the relation $R = \{(1, 1), (2, 1), (3, 2), (4, 3)\}$ on the set $S = \{1, 2, 3, 4\}$. 05
- ii. Let R be the relation on the set of real numbers such that xRy if and only if x and y are real numbers that differ by less than 1, that is $|x - y| < 1$. Determine whether or not the relation R is an equivalence relation. 05
- OR
- Q.9B Use Dijkstra's algorithm to find the length of shortest path between the vertices a and z in the weighted graph given in Q.7(A). 10