

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End-Term Examination, April 2024
B. Tech., Even Semester, 2023-24

Roll No.....

AS1116: Random Variables and Stochastic Processes

Time: 2 hours

Max. Marks: 30

Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. You may use a non-programmable scientific calculator.
3. Specify, if you find any ambiguity in the questions. Justify and solve the question after removing that ambiguity.

- Q.1 The amount of time needed for a certain machine to process a job is a random variable with mean $E(X_i) = 10$ minutes and $\text{Var}(X_i) = 2$ minutes². The time needed for different jobs is independent from each other. Find the probability that the machine processes less than or equal to 40 jobs in 7 hours. 5.0
(C02,3)
- Q.2 Let $\{X(t), t \in \mathbb{R}\}$ be a continuous-time random process, defined as 5.0
(C02,3)
$$X(t) = A \cos(2t + \Phi),$$
Where, $A \sim U(0, 1)$ and $\Phi \sim U(0, 2\pi)$ are two independent random variables.
a) Find the mean function $\mu_X(t)$.
b) Find the autocorrelation function $R_X(t_1, t_2)$.
c) Is $X(t)$ a wide-sense stationary (WSS) process?
- Q.3 Explain the random processes and their classifications. Provide a detailed explanation using some examples. Also, define what kind of random processes are called stochastic processes and highlight their significance in the real world. 5.0
(C02,4)
- Q.4 Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ . Find the probability that there are two arrivals in $(0, 2]$ or three arrivals in $(4, 7]$. 5.0
(C01,2)
- Q.5 Find the power spectral density function of a WSS process whose auto-correlation function is $R_X(\tau) = Ae^{-\lambda|\tau|}$. 5.0
(C03,5)
- Q.6 Consider the Markov chain in the following state transition diagram. Assume $X_0 = 4$. Find the expected time (number of steps) until the chain gets absorbed in R_1 or R_2 . More specifically, find $E[T | X_0 = 4]$, where T is the absorption time, i.e., the first time the chain visits a state in R_1 or R_2 . 5.0
(C03,4)

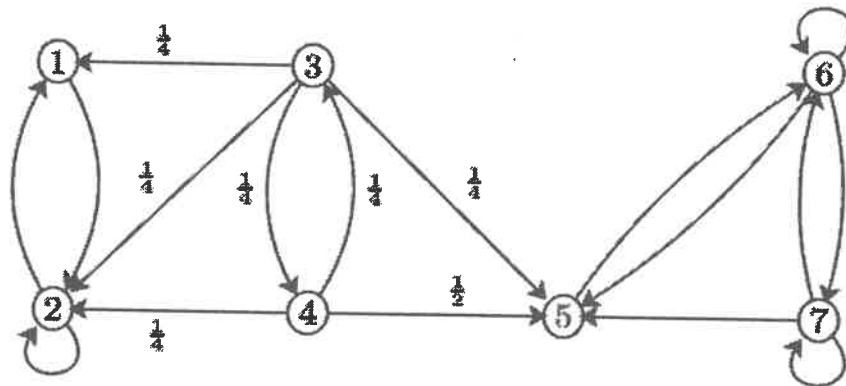
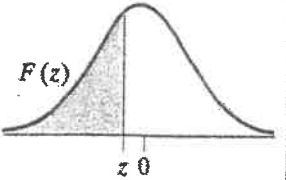


Table 3 Standard Normal Distribution Function

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-t^2/2} dt$$


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-5.0	0.0000003									
-4.0	0.00003									
-3.5	0.0002									
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0006	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

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