

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, December 2023

BTech, Semester III

Roll No.....

AS1110: DIFFERENTIAL EQUATIONS

Time: 3 hours

Max. Marks: 40

Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. Show all the steps properly in each answer.
3. Basic scientific calculator is allowed.

Q.1	a) Graph directional field for the differential equation $\frac{dy}{dx} = 1$. b) Give the order of differential equation, $e^{y'} + 3xy = 0$ and state whether the equation is linear or nonlinear. c) Will the set of functions, $\{\ln x, \ln x^2\}$ form a basis for the differential equation, $x^2y'' + xy' = 0$, when $x > 0$? d) What would be the choice for particular integral (y_p) for non-homogeneous differential equation, $y'' + ay' + by = r(x)$, using method of undetermined coefficients, if $r(x) = x^3e^x$. e) Find a linear ordinary differential equation (ODE), if the set of functions, $\{1, e^{2x}\}$ form a basis of the solution.	(5) (CO-1)
Q.2	a) Under what conditions for each of the constants, a, b, k, and l, is the differential equation exact? $(-ax + by)dx + (2kx + 2ly)dy = 0.$ b) Solve the given initial value problem and write the explicit form of the particular solution. $e^{2x}(-\sin y dy + 2 \cos y dx) = 0, y(0) = 0.$ c) Find the particular solution of the following differential equation and compute $y(2)$: $y' + \frac{3}{x}y = \frac{1}{x^3}, \quad y(1) = -1, \text{ where } x > 0.$	(2+4+4) (CO-1)
Q.3	a) Find the general solution of the Euler-Cauchy differential equation, $x^2y'' - 7xy' + 16y = 0.$ b) Solve the initial value problem $y^{(iv)} - 3y''' + 2y'' = 0, y(0) = 2, y'(0) = 0, y''(0) = 2, y'''(0) = 2.$	(3+4) (CO-1)

Q.4	a) Solve the following non-homogenous second order ODE using method of variation of parameters: $y'' + 3y' + 2y = 30e^{2x}.$ b) Apply the method of undetermined coefficients to solve the undamped mechanical system given by the non-homogeneous ODE: $(D^2 + 9I)y = 36 \cos(3x),$ where D is the differential operator and I is identity operator.	(4+4) (CO-1,3)
Q.5	Solve the following system of linear ODEs with given initial conditions using concept of eigenvalues: $\begin{aligned} y_1' &= 4y_2 \\ y_2' &= -4y_1 \\ y_1(0) &= 1, y_2(0) = 1. \end{aligned}$	(5) (CO-1)
Q.6	Consider the initial value problem, $y' - xy^2 = 0$, $y(0) = 1$. Use improved Euler's method with $h = 0.1$ to obtain an approximation to $y(0.3)$. Calculate the absolute error at each step if the analytical solution is given as $y(x) = e^{x^2/2}$.	(5) (CO-2)

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