

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, December 2023

B. Tech., Semester I, 2023-24

Roll No.....

AS1109: CALCULUS

Time: 3 hours

Max. Marks: 40

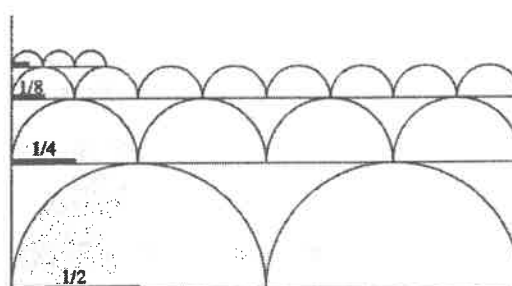
Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. Mention all the assumptions for your answers clearly.

Q. 1.1 Find the values of x for which the geometric series $\sum_{n=0}^{\infty} (\ln x)^n$ converges.

1.0
(CO1)

Q. 1.2 The figure given below shows the first three rows and part of the fourth row of a sequence of semicircles. There are 2^n semicircles in the n th row, each of radius $\frac{1}{2^n}$. Find the sum of the areas of all semicircles.



2.0
(CO1)

Q. 1.3 Construct an example to show that $\sum_{n=1}^{\infty} a_n b_n$ may diverge even if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge.

2.0
(CO1)

Q. 2.1 Find the first order partial derivative $\frac{\partial f}{\partial x}$ for the function $f(x, y) = \sum_{n=0}^{\infty} (xy)^n$, $|xy| < 1$.

1.0
(CO1,2)

Q. 2.2 At what points (x, y) in the plane is the function $f(x, y) = \frac{x^2 + y^2}{x^2 - 5x + 6}$ continuous?

1.0
(CO2)

Q. 2.3 By considering different paths of approach, show that the limit of the function $f(x, y) = \frac{y}{x^2 - y}$ does not exist as $(x, y) \rightarrow (0, 0)$.

2.0
(CO2)

Q. 2.4 Show that if we substitute $x = r \cos \theta$ and $y = r \sin \theta$ in a differentiable function $w = f(x, y)$, then

4.0
(CO2,3)

$$\frac{\partial w}{\partial r} = f_x \cos \theta + f_y \sin \theta \quad \text{and} \quad \frac{1}{r} \frac{\partial w}{\partial \theta} = -f_x \sin \theta + f_y \cos \theta.$$

Further, show that

$$(f_x)^2 + (f_y)^2 = \left(\frac{\partial w}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta} \right)^2.$$

Q. 3.1 Determine the isotherms (curves of constant temperature T) of the temperature fields in the plane given by the scalar function $T(x, y) = xy$. Sketch some isotherms.

1.0
(CO2,3)

Q. 3.2 Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$, $0 \leq x \leq 1$.

3.0
(CO3)

Q. 3.3 Sketch the region of integration and evaluate the integral $\int_0^{\pi} \int_0^{\sin x} y \, dy \, dx$.

3.0
(CO3)

Q. 3.4 Set up an integral to find the volume of the solid cut from the square column $|x| + |y| \leq 1$ by the planes $z = 0$ and $3x + z = 3$. **Don't compute the integral.**

1.0
(CO3)

- Q. 3-5 Change the Cartesian integral $\int_0^6 \int_0^y x \, dx \, dy$ into an equivalent polar integral. Then evaluate the polar integral. 3.0
(CO3)
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- Q. 4.1 For the vector field $\vec{v} = [x^2y^3 - z^4, 4x^5y^2z, -y^4z^6]$, compute $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{v})$. 1.0
(CO2,4)
- Q. 4.2 Figures below show the sketch of four different vector fields. Classify them as vector field with positive, negative, and zero divergence. 1.0
(CO4)
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- Q. 4.3 Find the directions in which the function $f(x, y, z) = xe^y + z^2$ increases and decreases most rapidly at $P(1, \ln 2, \frac{1}{2})$. Find the derivative of the function in these directions. 3.0
(CO4)
- Q. 4.4 Find the (unit) directions $\vec{u}$ in which the derivative of the function $f(x, y) = x^2 - xy + y^2 - y$ at point $P(1, -1)$ is zero. 2.0
(CO4)
- Q. 4.5 Given the Laplace of the function f , $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$, compute $\nabla^2 f$, for $f(x, y, z) = 2z^3 - 3(x^2 + y^2)$. 2.0
(CO2,4)
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- Q. 5.1 Determine the truth values (T/F) of the following conditional statements:
a. n is even if and only if $(3n + 2)$ is even.
b. If an integer n is prime, then n is odd. 1.0
(CO5)
- Q. 5.2 What is wrong with this famous supposed “proof” that $1 = 2$? Explain. 1.0
(CO5)
- Step 1. $a = b$
Step 2. $a^2 = ab$
Step 3. $a^2 - b^2 = ab - b^2$
Step 4. $(a + b)(a - b) = b(a - b)$
Step 5. $a + b = b$
Step 6. $2b = b$
Step 7. $2 = 1$
- Q. 5.3 Use a direct proof to show that every odd integer is the difference of two squares. 1.0
(CO5)
- Q. 5.4 Let p and q be the propositions. p : You drive over 65 miles per hour. q : You get a speeding ticket. Write these propositions using p and q and logical connectives (including negations). 4.0
(CO5)
- a. You drive over 65 miles per hour, but you do not get a speeding ticket.
b. You will get a speeding ticket if you drive over 65 miles per hour.
c. If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
d. Driving over 65 miles per hour is sufficient for getting a speeding ticket.
e. You get a speeding ticket, but you do not drive over 65 miles per hour.
f. Whenever you get a speeding ticket, you are driving over 65 miles per hour.
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