

JK LAKSHMIPAT UNIVERSITY, JAIPUR**INSTITUTE OF ENGINEERING AND TECHNOLOGY**

End Term Examination (Regular/Backlog), Nov - Dec, 2015

B. Tech.(All Branches), Semester III

Roll No.....

COURSE CODE: MA 301

Time: 3 hours

Max. Marks: 100

COURSE TITLE: Engineering Mathematics III

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Calculators are allowed.

Q.1A Objective type multiple choice questions (10 questions @ 1 mark)

10

i. Function $f(z) = \sin\left(\frac{1}{z}\right)$ has type of singularity at $z=0$

- (a) removable (b) essential (c) pole (d) zero

ii. If $P_n(x)$ is the Legendre polynomial of order n , then $3x^2+3x+1$ can be expressed as

- (a) $2P_2+3P_1$ (b) $4P_2+2P_1+P_0$
 (c) $3P_2+3P_1+P_0$ (d) $2P_2+3P_1+2P_0$

iii. Which of the following is true for $f(z) = \frac{1}{z}$

- (a) Analytic everywhere (b) nowhere analytic
 (c) Analytic only at Origin (d) Analytic everywhere except at Origin

iv. Which Fourier transform can be used to solve the following BVP

$$K \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad -\infty < x < \infty, t > 0$$

$$s.t. u(x, 0) = f(x), \quad \text{where } f(x) = \begin{cases} u_0 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

- (a) Fourier sine transform (b) Fourier cosine transform
 (c) complex Fourier transform (d) all of these

v. $\int_0^{\infty} F_s(\alpha) \frac{\sin \alpha x}{a^2 + b^2} d\alpha = \dots$, where $F_s(\alpha)$ denotes the Fourier sine transform of $f(x)$ w.r.t. parameter α .

- (a) $f(x)$ (b) $\frac{\pi}{2(a^2 + b^2)} f(x)$ (c) $\frac{\pi}{2} f(x)$ (d) none of these

vi The Laplace transform of $f(t) = t^2 e^{-t}$ is

- (a) $\frac{2}{(s+1)^3}$ (b) $\frac{-2}{(s+1)^3}$ (c) $\frac{-2}{(s-1)^3}$ (d) $\frac{2}{(s-1)^3}$

vii The triangle having the smallest perimeter for a given base and given area will be

- (a) Equilateral (b) isosceles (c) Rt. Angle triangle (d) can't say

viii Which of the following is true regarding zeros of the function $f(z) = \frac{1 - e^z}{(z^2 - 4)^3}$

- (a) zero of order 3 at $z=2$ (b) zero of order 3 at $z=-2$
 (c) simple zero at $z=0$ (d) both (a) and (b)

ix The residue of the function $f(z) = \frac{\sin 2z}{z^2}$ about its pole $z=0$, is

- (a) 1 (b) 2 (c) $4/3$ (d) zero

x The inverse Laplace transform of $\frac{1}{(2s-7)}$

- (a) e^{-7t} (b) e^{7t} (c) $\frac{1}{2} e^{\frac{7t}{2}}$ (d) $\frac{1}{2} e^{-\frac{7t}{2}}$

Q.2 Short answer type questions (4 questions @ 2.5 marks)

10

(i) Explain isoperimetric problems. Also give the necessary condition for integral

$$\int_{x_1}^{x_2} f(x, y, y') dx \text{ to be extremum.}$$

(ii) Define conformal mapping. Check whether magnification ($w=cz$) is a conformal mapping or not.

- (iii) Find the Laplace transform of the periodic function

$$f(t) = e^t \text{ for } 0 < t < 2\pi$$

- (iv) Find the Fourier sine transform of
- $f(x) = e^{-ax}$
- .

Q.3A If $L\{\sin \sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-\frac{1}{4s}}$, then find the Laplace transform of $L\left\{\frac{\cos \sqrt{t}}{\sqrt{t}}\right\}$. 05

Q.3B Prove that $u = x^2 - y^2 - 3xy - 2x + 3y$ is harmonic. Find the harmonic conjugate of u and hence form the analytic function $f(z)$. 05

Q.4A Evaluate the following integral using Cauchy integral formula 05

$$\int_c \frac{1}{z(z-1)(z-2)} dz, \text{ where } c \text{ is the circle } |Z+1| = 3.$$

Q.4B Using Residue theorem, evaluate $\int_c \frac{e^{zt}}{z^2(z^2+2z+2)} dz$, where c is the circle $|Z| = 3$. 05

Q.5A Solve the following initial value problem using Laplace transform technique 05

$$(D^2 + D - 2)y = 1 - 3x, \text{ at } x=0, y=0 \text{ and } \frac{dy}{dx} = 4.$$

Q.5B Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. 05

Q.6A Prove that: $x \cos x = 2[1^2 J_1 - 3^2 J_3 + 5^2 J_5 - \dots]$, where $J_n(x)$ denotes the Bessel function of n^{th} order. 05

Q.6B Find the inverse Laplace transform of $F(s) = \frac{16}{(s+2)^2(s-2)}$ by convolution theorem. 05

OR

Q.6B If the sine transform of $f(x)$ is $\frac{e^{-s}}{s}$, then find $f(x)$. 05

Q.7A Find the Taylor's expansion of $f(z) = \frac{1}{z^2 - z - 12}$ about (a) $z=-1$, (b) $z=0$. 05

Q.7B Establish the following 05

(i) $J_3(x) + 3J_0'(x) + 4J_0'''(x) = 0$

(ii) $P_n'(x) - xP_{n-1}'(x) = nP_{n-1}(x)$

OR

Q.7B Find the surface with the smallest area which encloses a given volume. 05

Q.8A Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$, where $P_n(x)$ denotes the Legendre polynomial of order n . 10

Q.9A Using contour integration, evaluate the real integral $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos \theta} d\theta$. 10

Q.9B A string is stretched with fixed end points $x=b$ and $x=c$ is initially in a position given by $y = b \sin\left(\frac{\pi x}{c}\right)$. It is released from rest in this position. If y satisfies the equation $\frac{\partial^2 y}{\partial x^2} = a^2 \frac{\partial^2 y}{\partial t^2}$, then show by the method of Fourier transform that the displacement y at any distance x from one end and at any time t is given by

$$y = b \sin\left(\frac{\pi x}{c}\right) \cos\left(\frac{\pi a}{c} t\right)$$

OR

Q.9B Solve the following system of simultaneous differential equations using Laplace transform 10

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t; \quad x(0)=1, y(0)=0.$$