



JK Lakshmipat University, Jaipur
Institute of Engineering and Technology
End Term Examination

B.Tech. First Year/First Semester/2013-2014

Course Name: Engineering Mathematics-I

Course Code: MA 101

Max Marks: 40

Time: 03.00 hrs

Instructions: Use of calculator is not permitted. Be Neat! Be Legible!! All the Best!!!

Section A (Attempt all questions)

Q.1. (a) Define Orthogonal function. Also, write Fourier-Euler formulae.

(b) Find the unit normal vector for $r(t) = t\hat{i} + t^2\hat{j}$ when $t=1$.

(c) Improper integrals are (i), and (ii)

(d) A function $f(x,y)$ is said to be homogenous function if each term has distinct power. (T/F)

(e) Gauss divergence theorem transforms line integrals to volume integrals and conversely. (T/F)

(1 × 5 = 5)

Q.2. (a) Find the absolute maximum and minimum values of

$$f(x, y) = 2 + 2x + 2y - x^2 - y^2$$

on triangular plate in the first quadrant by lines $x=0$, $y=0$ and $y=9-x$.

(b) Explain: (i) Tangent Plane, and (ii) Arc Length.

(c) Find all the asymptotes of the curve $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.

2½
½+½

1½

Section B (Attempt any four questions)

Q.3. If $u=f(r)$ and $x = r \cos \theta$, $y = r \sin \theta$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

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Q.4. Given $\int_0^\infty \frac{x^{n-1}}{1+x} dx = \frac{\pi}{\sin n\pi}$, $0 < n < 1$, show that, $\Gamma n \Gamma n - 1 = \frac{\pi}{\sin n\pi}$.

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Q.5. Trace the strophoid $y^2(x+a) = x^2(3a-x)$.

5

Q.6. Obtain the Fourier series of $f(x) = (\pi - x)/2$ in the interval $(0, 2\pi)$. Deduce that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

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Q.7. (a) Discuss the convergence of the series

$$\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \dots$$

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(b) Evaluate $\iiint_D xyz \, dx \, dy \, dz$, where D is the region enclosed by the ellipsoid

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1.$$

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Section C

Q.8. (a) Find the area bounded by the curves $y^2 = x^3$ and $x^2 = y^3$.

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(b) If $u = x^3 y^2 \sin^{-1}(y/x)$ show that

$$xu_x + yu_y = 5u \text{ and } x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 20u.$$

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Or

Q.8. (a) Use divergence theorem to evaluate the surface integral: $\iint_S yz \, dy \, dz + xz \, dz \, dx + xy \, dx \, dy$

where $S: x^2 + y^2 + z^2 = 4$.

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(b) If f and g are solutions of the Laplace equation, then show that

$$\nabla \cdot (f \nabla g - g \nabla f) = 0.$$

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