

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, December 2022

BTech, Semester V

Roll No.....

AS1404: FIRST COURSE ON PARTIAL
DIFFERENTIAL EQUATION

Time: 3 hours

Max. Marks: 40

Instructions to students:

1. Do not write anything other than your roll number on the question paper.
 2. All questions are compulsory.
 3. Throughout the question paper, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, and $t = \frac{\partial^2 z}{\partial y^2}$.
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- Q.1A** Write the general form of second-order linear partial differential equations (x and y being independent variables and z being a dependent variable). 1.0
(LO-2)
- Q.1B** Form a partial differential equation by eliminating arbitrary constants a, b and c from the following relation: 5.0
(LO-1,2)
- $$z = ax^2 + bxy + cy^2.$$
- Also, state the order and degree of the partial differential equation obtained.
- Q.1C** Describe the fundamental classification of the second-order linear partial differential equations and hence, classify the following differential equations with proper justification. Also, mention the domain of classification: 8.0
(LO-2)
- a. $\frac{\partial^2 u}{\partial x^2} + 4 \left(\frac{\partial^2 u}{\partial x \partial y} \right) + 4 \left(\frac{\partial^2 u}{\partial y^2} \right) = 0$
- b. $y \left(\frac{\partial^2 z}{\partial x^2} \right) + x \left(\frac{\partial^2 z}{\partial y^2} \right) = 0$
- Q.2A** True or False: 1.0
(LO-1,2)
- Singular solution of a differential equation is one that cannot be obtained from the general solution acquired by the usual method of solving the differential equation.
- Q.2B** Show that if f and g are arbitrary functions, then $z = f(x^2 - y) + g(x^2 + y)$ is a solution of the following partial differential equation: 5.0
(LO-1,2)
- $$\frac{\partial^2 z}{\partial x^2} - \frac{1}{x} \frac{\partial z}{\partial x} = 4x^2 \frac{\partial^2 z}{\partial y^2}.$$
- Q.2C** Find the complete integral of the following first-order linear partial differential equations: 8.0
(LO-2)
- a. $x(1 + y)p = y(1 + x)q$
- b. $pxy + pq + qy = yz.$

- Q.3A** Find all eigenvalues and their corresponding eigenfunctions of the two-point boundary value problem: 5.0
(LO-2)

$$X'' + \lambda X = 0, X'(0) = 0 \text{ and } X(2\pi) = 0,$$

where X is the function of space variable, x . (Check all the three cases)

- Q.3B** Find the temperature distribution, $u(x, t)$ in a laterally insulated heat conducting bar of length 2, whose ends are kept at temperature zero and the initial temperature in the bar is $u(x, 0) = \sin\left(\frac{\pi x}{2}\right) - 3 \sin(2\pi x)$. Consider thermal diffusivity of this thin homogeneous bar as 3. (Write the mathematical model of the problem and then solve it using appropriate method). 7.0
(LO-2)
