

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Backlog), Nov. 2018

B. Tech.(All Branches), Semester II

Roll No.....

COURSE CODE: MA 202

Time: 3 hours

Max. Marks: 100

COURSE TITLE: Differential Equations and Complex Analysis

Instructions to students:

1. Do not write anything other than your roll number on question paper. Be neat and clean!!!!

Q.1A Objective type multiple choice questions (10 questions @ 1 mark)

- i. An integrating factor transforms first-order differential equation to
 - (a) Bernoulli's form
 - (b) a homogeneous form
 - (c) a linear form
 - (d) exact form
- ii. The C.F. of the ODE $(D^2 - a^2)^2 y = x \cos x e^x$.
 - (a) $(C_1 + C_2 x) \sin x + (C_3 - C_4 x) \cos x$
 - (b) $C_1 e^x + C_2 e^{-x}$
 - (c) $(C_1 + C_2 x) e^x + (C_3 + C_4 x) e^{-x}$
 - (d) $(C_1 \pm C_2 x) e^{\pm x}$
- iii. Which of the following is true for $f(z) = z^2$
 - (a) Analytic everywhere
 - (b) nowhere analytic
 - (c) Analytic only at Origin
 - (d) Analytic everywhere except at Origin
- iv. The steady state temperature distribution in a solid without a heat source can be expressed as:
 - (a) Poisson's equation
 - (b) Laplace equation
 - (c) diffusion equation
 - (d) wave equation
- v. Function $f(z) = \sin\left(\frac{1}{z}\right)$ has type of singularity at $z=0$
 - (a) removable
 - (b) essential
 - (c) pole
 - (d) zero
- vi. The inverse Laplace transform of $\frac{3}{(s^2 - 9)}$
 - (a) $\sin 3t$
 - (b) $\cos 3t$
 - (c) $\sinh 3t$
 - (d) $\cosh 3t$
- vii. If f is analytic at a point and in its neighbourhood, then
 - (a) f is an entire function
 - (b) f is harmonic function
 - (c) f is differentiable at that point
 - (d) None of them
- viii. The Laplace transform of $\frac{1}{\sqrt{t}}$ is
 - (a) not defined
 - (b) $\sqrt{\frac{\pi}{s}}$
 - (c) $\frac{\pi}{\sqrt{s}}$
 - (d) $\left(\frac{1}{2}\right)! s^{-\frac{1}{2}}$
- ix. If $f(x) = e^x$, $-\pi < x < \pi$, then a_0 is
 - (a) $\frac{2}{\pi} \sin h\pi$
 - (b) $\frac{1}{\pi} \sin h\pi$
 - (c) $2 \sin h\pi$
 - (d) $\frac{4}{\pi} \sin h\pi$
- x. Laplace transforms provides a mathematical framework for the conversion of a function or derivative of a function from
 - (a) s domain to t domain in an algebraic form

- (b) t domain to s domain in an integral form
- (c) t domain to s in an algebraic form
- (d) s domain to t domain in an integral form

Q.2 Short answer type questions (4 questions @ 2.5 marks) 10

Q.2A Solve: $\frac{dy}{dx} + y = y^2$

Q.2B Find the Laplace transform of the periodic function
 $f(t) = e^t$ for $0 < t < 2\pi$

Q.2C Determine (a) type, (b) order, (c) dependent variables and (d) independent variable of the PDE $\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$.

Q.2D Determine the poles of the following function and residue at each pole

$$f(z) = \frac{z}{(z+1)^2(z-2)}.$$

Q.3A Find the complete solution of the second order ODE 05

$$(D^2 + 2)y = e^{3x} + \sin x.$$

Q.3B Solve the heat equation 05

$$K \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad -\infty < x < \infty, t > 0$$

$$s.t. u(x, 0) = f(x), \text{ where } f(x) = \begin{cases} u_0 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

Q.4A Solve: $\frac{dy}{dx} + y = y^2$ 05

Q.4B Solve: $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$ 05

Q.5A Find Fourier sine transform of as $f(x) = \begin{cases} x, & 0 < x \leq 1 \\ -x, & 1 < x < 2 \\ 0, & x \geq 2 \end{cases}$ 05

Q.5B Solve: $(D^2 + 2D + 1)y = e^x \sin x$ 05

Q.6A Prove that $u = x^2 - y^2 - 3xy - 2x + 3y$ is harmonic. Find the harmonic conjugate of u and hence form the analytic function f(z). 05

Q.6B Find the inverse Laplace transform of $\frac{16}{(s-2)(s+2)^2}$. 05

Q.7 Apply Laplace transform to solve the following ODE $\frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} - 8y = 0$; $y(0)=0=y'(0)$. 10

OR

Q.7 In a transmission line, a dc signaling voltage is applied and the following relation is observed $\frac{d^2 v}{ds^2} - \alpha^2 v = 0$, where v is a voltage at a point on the line, s is the distance of the point from the end of the line, and α is the attenuation constant of the line. Find a relation for v and express it in hyperbolic form. 10

Q.8 Write the statement of Residue theorem. Then, apply this theorem to solve 10

$\int_C \frac{1}{z(z-1)(z+3)} dz$ where c is the circle $|z| = 2$.

Q.9A Solve the following system of simultaneous differential equations using Laplace transform 10

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t; \quad x(0)=1, y(0)=0.$$

Q.9B Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, in rectangle 10

$$u(x, 0)=f(x), \quad u(x, b)=0 \quad U(0,y)=0, \quad u(a, y)=0.$$

OR

Q.9B Explain even and odd functions. Then, discuss the Fourier series. Find the 10

$$\text{Fourier series for the function defined as } f(x) = \begin{cases} -1, & \pi \leq x < 0 \\ +1, & 0 < x \leq \pi \end{cases}$$