

Heat and Mass Transfer in MHD Three Dimensional Free Convective Flow Past a Vertical Surface with Periodic Suction

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Abstract

Three-dimensional mixed convection flow past a vertical plate under the combined buoyancy effects of thermal and mass diffusion has been studied under the action of transverse applied magnetic field taking into account the transverse sinusoidal suction at the plate, when the plate is subjected to constant heat and mass fluxes. Solutions for the velocity field, temperature distribution, and concentration distributions are obtained. The effect of magnetic field parameter, suction parameter and Schmidt number on the flow variables has been analyzed.

1 INTRODUCTION

Many transport processes can be found in various ways in both nature and technology, in which the combined heat and mass transfer occur due to buoyancy forces caused by thermal diffusion (temperature differences) and mass diffusion (concentration differences). Some of the convective heat and mass transfer processes with phase change include the evaporation of a liquid at a interface between a gas and a liquid or the sublimation at a gas-solid interface. These can be described using the methods for convective heat and mass transfer. Separation processes in chemical engineering, distillation, extraction and absorption are all affected by the process of mass transfer. The problem of steady mixed convection adjacent by a heated vertical flat plate has received a great deal of attention in past (Dey and Nath [1]). Kaware and Vibrechi [2] investigated the free convection flow past an infinite vertical plate with constant suction. Martynenko et. al. [3] investigated the free convection flow past an infinite vertical plate with constant suction. Pantokratoras [4] has studied the laminar free convection flow over a vertical plate with uniform injection or suction. Natural free convection with mass transfer on an isothermal flat plate was studied by Schenk et. al. [5] and Gallahan and Marner [6]. Soundalgekar and Warve [7] analyzed two dimensional unsteady free convection flow in the presence of foreign mass, past an infinite vertical plate. Soundalgekar and Ganesan [8] further discussed the above problem by using a finite difference scheme. An extensive contribution on heat and mass transfer flows has been made by Khair and Bejan [9]. Lin and Wu [10] have analyzed the problem of simultaneous heat and mass transfer with entire range of buoyancy ratio for the most practical chemical species in dilute and aqueous solutions. Singh et. al. [11] studied the three dimensional mixed convection flow past a vertical plate with periodic suction.

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Magnetohydrodynamics (MHD) is currently undergoing a period of great enlargement and differentiation of subject matter. The interest in these new problems originates from their importance in liquid metals, electrolytes and ionized gases. The MHD heat and mass transfer processes are of interest in power engineering, metallurgy, astro-physics and geophysics. Hydromagnetic heat and mass transfer in flow of a viscous incompressible fluid past an infinite vertical porous plate under oscillatory suction normal to the plate was investigated by Singh *et. al.* [12]. The problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface was analyzed by Chen [13]. Chaudhary and Sharma [14] studied the heat and mass transfer flow of viscous electrically conducting fluid past a vertical surface, taking induced magnetic field into account. Chaudhary *et. al.* [15] studied heat and mass transfer in MHD mixed convection flow past a vertical surface taking radiation effect into account.

The problem investigated here is the heat and mass transfer flow of a three-dimensional viscous electrically conducting fluid past a vertical surface under the action of a uniform transverse magnetic field with constant heat and mass flux (CHF / CMF) and periodic suction.

2 MATHEMATICAL FORMULATION

The sinusoidal suction velocity distribution at the plate is considered to be of the form

$$\bar{v}(\bar{z}) = -V_0 \left(1 + \epsilon \cos \frac{\pi U_0 \bar{z}}{v} \right)$$

which consists of a basic distribution $V_0 > 0$ superimposed with a very weak distribution $\epsilon \cos \frac{\pi U_0 \bar{z}}{v}$. Therefore the amplitude πU_0 of the suction velocity variation is assumed to be small. We chose a co-ordinate system with plate lying vertically on $\bar{x} - \bar{z}$ plane such that $\bar{x}$ is taken along the plate in the direction of the flow and $\bar{y}$ axis is perpendicular to the plate. The magnetic field is applied along the $\bar{y}$ axis. Denoting velocity components $\bar{u}, \bar{v}$ and $\bar{w}$ in the direction $\bar{x}, \bar{y}$ and $\bar{z}$ and temperature, concentration and pressure by $\bar{T}, \bar{C}$ and $\bar{p}$ respectively, the flow is governed by the following equations under usual Boussinesq's approximation:

$$\bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{v}}{\partial \bar{z}} = 0 \quad \dots(1)$$

$$\bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} = g\beta(\bar{T} - \bar{T}_\infty) + g\beta^*(\bar{C} - \bar{C}_\infty) + \frac{\sigma B_0^2}{\rho'}(u_0 - \bar{u}) + \nu \left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) \quad \dots(2)$$

$$\bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{v}}{\partial \bar{z}} = -\frac{1}{\rho'} \frac{\partial \bar{p}}{\partial \bar{y}} + \nu \left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right) \quad \dots(3)$$

$$\bar{v} \frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho'} \frac{\partial \bar{p}}{\partial \bar{z}} + \nu \left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right) - \frac{B_0^2 \bar{w}}{\rho'} \quad \dots(4)$$

$$\bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{T}}{\partial \bar{z}} = \kappa \left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right) \quad \dots(5)$$

$$\bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{C}}{\partial \bar{z}} = D \left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2} \right) \quad \dots(6)$$

where ν is the kinematic viscosity, g the acceleration due to gravity, β the volumetric coefficient with thermal expansion, β^* the volumetric coefficient of expansion with concentration, B_0 the magnetic field, $\bar{T}_\infty$ and $\bar{C}_\infty$ are the temperature and concentration respectively far away from the plate, $\bar{p}$ the pressure near the plate, σ the scalar electrical conductivity, ρ the density, κ the thermal conductivity.

The plate being infinite in length hence the flow variables are functions of $\bar{y}$ and $\bar{z}$ only. The boundary conditions of the problem are:

$$\begin{aligned} \bar{y} = 0: \bar{u} = 0; \bar{v} = -V_0 \left(1 + \epsilon \cos \frac{\pi U_0 \bar{z}}{v} \right); \bar{w} = 0; \frac{\partial \bar{T}}{\partial \bar{y}} = -\frac{Q}{\kappa}; \frac{\partial \bar{C}}{\partial \bar{y}} = -\frac{j}{D} \\ \bar{y} \rightarrow \infty: \bar{u} = U_0; \bar{v} = -V_0; \bar{w} = 0; \bar{p} = \bar{p}_\infty; \bar{T} = \bar{T}_\infty; \bar{C} = \bar{C}_\infty \end{aligned} \quad \dots(7)$$

Where $\bar{p}_\infty$ is the pressure far away from the plate; U_0 is the free stream velocity; Q is the velocity constant heat flux per unit area and j is the constant mass flux per unit area.

Converting into non-dimensional form, the equations (1) to (6) become:

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = 0 \quad \dots(8)$$

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = G_r \theta + G_c C + M(1 - u) + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad \dots(9)$$

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \quad \dots(10)$$

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} - Mw \quad \dots(11)$$

$$v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad \dots(12)$$

$$v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = \frac{1}{Sc} \left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \quad \dots(13)$$

where G_r - the thermal Grashoff number, G_c - the mass Grashoff number, Pr - the Prandtl number, M - The Hartmann number, Sc - the Schmidt number and λ is the suction parameter. The corresponding boundary conditions are reduced to

$$\begin{aligned} y = 0: u = 0, v = -\lambda(1 + \epsilon \cos \pi z) \\ w = 0: \frac{\partial \theta}{\partial y} = -1; \frac{\partial C}{\partial y} = -1 \\ y \rightarrow \infty: u = 1, v = -\lambda; w = 0; p = p_\infty; \theta = 0; C = 0 \end{aligned} \quad \dots(14)$$

3 SOLUTION OF THE PROBLEM

When the amplitude $\epsilon \ll 1$, we assume the solution in the neighborhood of the plate in the form of series in terms of increasing powers of $u = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \dots$, and similar series for v, w, θ and C . Substituting these series in equations (8) to (14), equating the coefficients of like powers of ϵ , and neglecting the coefficients of ϵ^2 , we get and further applying transformed boundary conditions in the terms free from ϵ , we get

$$u_0 = (K_1 + K_2 - 1)e^{-m_1 y} + 1 - K_1 e^{-\lambda P r y} - K_2 e^{-\lambda S c y} \quad \dots(15)$$

$$\theta_0 = \frac{e^{-\lambda P r y}}{\lambda P r} \quad \dots(16)$$

$$C_0 = \frac{e^{-\lambda S c y}}{\lambda S c} \quad \dots(17)$$

where K_1 and K_2 are constants. with transverse velocity components $v_0 = -\lambda$, $w_0 = 0$, and the pressure $p_0 = p_\infty$. This is the solution of a steady two – dimensional problem with constant suction at the vertical plate.

Taking into account the solutions of the transverse velocity components $v_0 = -\lambda$, $w_0 = 0$ the terms as the coefficient of ε under the transformed boundary conditions are

$$v_1(y, z) = (-K_3 e^{-r_1 y} + K_4 e^{-r_2 y}) \cos \pi z \quad \dots(18)$$

Using this result, w_1 is obtained as

$$w_1(y, z) = -\frac{1}{\pi} (K_3 r_1 e^{-r_1 y} - K_4 r_2 e^{-r_2 y}) \sin \pi z \quad \dots(19)$$

and

$$\theta_1(y, z) = (K_7 e^{-r_3 y} + K_5 e^{-(\lambda Pr + r_1)y} - K_6 e^{-(\lambda Pr + r_2)y}) \cos \pi z \quad \dots(20)$$

$$C_1(y, z) = (K_{10} e^{-r_5 y} + K_8 e^{-(\lambda Sc + r_1)y} - K_9 e^{-(\lambda Sc + r_2)y}) \cos \pi z \quad \dots(21)$$

$$u_1(y, z) = (K_{19} e^{-r_1 y} - K_{11} e^{-r_3 y} - K_{12} e^{-r_5 y} - K_{13} e^{-(\lambda Pr + r_1)y} + K_{14} e^{-(\lambda Pr + r_2)y} - K_{15} e^{-(\lambda Sc + r_1)y} + K_{16} e^{-(\lambda Sc + r_2)y} + K_{17} e^{-(m_1 + r_1)y} - K_{18} e^{-(m_1 + r_2)y}) \cos \pi z \quad \dots(22)$$

where K_3 to K_{19} and r_1 to r_5 are constants.

Skin-Friction

The skin friction is expressed as

$$\tau_x = \left\{ \frac{du_0}{dy} \right\}_{y=0} + \varepsilon \left\{ \frac{du_{11}}{dy} \right\}_{y=0} \cos \pi z = K_{20} + \varepsilon K_{21} \cos \pi z \quad \dots(23)$$

and

$$\tau_z = \varepsilon \left\{ \frac{dw_{11}}{dy} \right\}_{y=0} \sin \pi z = \varepsilon K_{22} \sin \pi z \quad \dots(24)$$

where K_{20} to K_{22} are constants.

4 DISCUSSION AND CONCLUSION

Combined heat and mass transfer effects on three-dimensional free convective flow of viscous incompressible fluid flow past a vertical plate in the presence of transverse magnetic field has been carried out in the preceding sections. In order to get physical insight in the problem, the velocity, temperature, concentration fields, and skin friction have been discussed by assigning numerical values to M (Magnetic field parameter), λ (Suction parameter), Pr (Prandtl number), Sc (Schmidt number), Gr (Grashoff number) and Gc (Modified Grashoff number).

Figure 1 depicts the velocity profiles for different values of M (Magnetic field parameter) and λ (Suction parameter). It is observed that an increase in M (Magnetic field parameter) leads to decrease the velocity i.e., the magnetic field exerts restraining force on the fluid, which tend to impede its motion. The velocity decreases with increasing values of λ . The effects of Gr (Grashoff number) and Gc (Modified Grashoff number) in water ($Pr = 0.71$) and air ($Pr = 7.0$) are shown in figure 2 and figure 3 respectively. It is clearly observed from figure 2 that velocity increases with increasing values of Gr in water but fluid velocity is uniform in air being unaffected by alteration in Gr . Figure 3 shows an increase in velocity with increasing Gr in both the medium. It is also found from figure 2 and figure 3 that fluid velocity decreases with increasing values of Pr (Prandtl number). The effect of Sc (Schmidt number) on velocity is depicted

in figure 4 where a decrease in velocity is found with increasing Sc . In addition, the curves in figure 2 to 4 show that the peak value of the velocity increases rapidly near the wall of the porous plate and then decays to the free stream velocity.

The concentration profile for different values of λ (Suction parameter) and Sc (Schmidt number) is investigated in figure 5 and found that an increase in λ and Sc results in decreasing concentration i.e., the concentration for the lighter particle is more than that of the heavier particle.

The variations of the temperature profiles for different values of Pr (Prandtl number) have been shown in figure 6. The results show that an increase in Prandtl number results a decrease in the thermal boundary layer. The reason is that the smaller values of Pr are equivalent to increasing the thermal conductivities and therefore heat is able to diffuse away from the heated surface more rapidly than for larger values of Pr . Hence the boundary layer is thicker and rate of heat is reduced.

The skin-friction in the main-flow direction is presented in figure 7 for different values of Gr , Gc , Sc , λ and Pr . It is observed that the skin-friction increases with the increase in Gr (Grashoff number) and Gc (modified Grashoff number) but increase due to Gr is more than the increase in Gc . It is also observed that skin-friction decrease for increasing values of Pr (Prandtl number), λ (suction parameter) and Sc (Schmidt number). Therefore it is concluded that the skin-friction for lighter particle is more than that of the heavier particle i.e., the skin-friction increases owing to greater cooling of the plate. It is also concluded that for $0.5 \leq z \leq 1.5$, the values of skin-friction is far less in water than those in the case of air but the reversal phenomenon occurs for $1.5 \leq z \leq 2.5$. Sinusoidal nature of skin-friction is also observed.

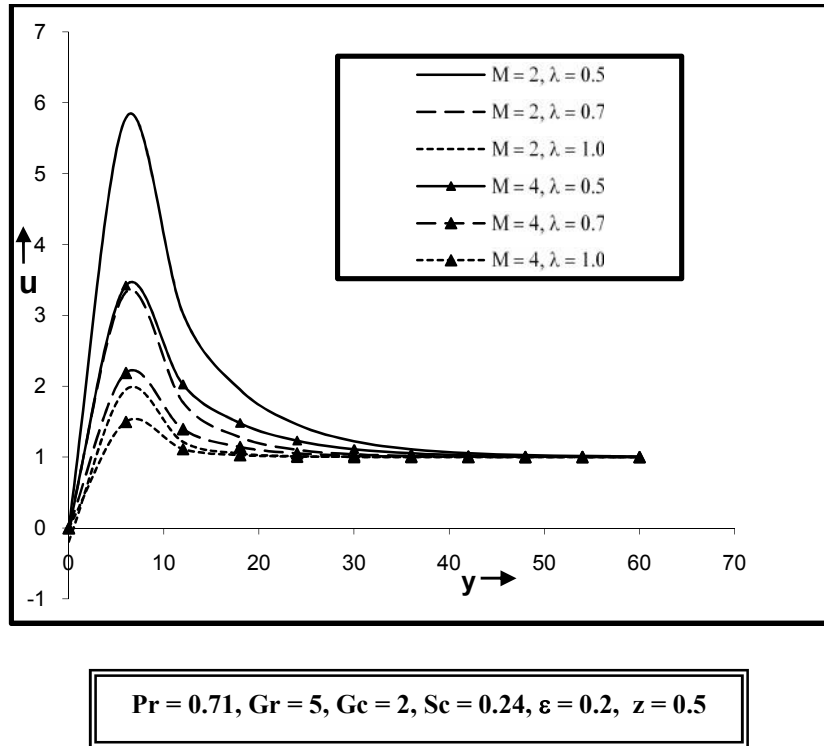
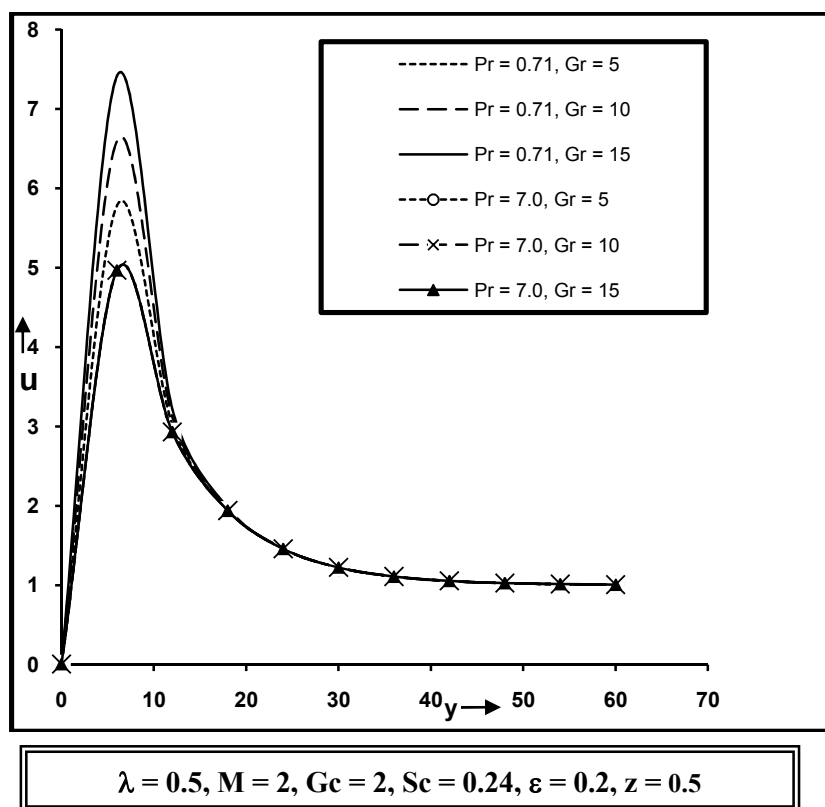
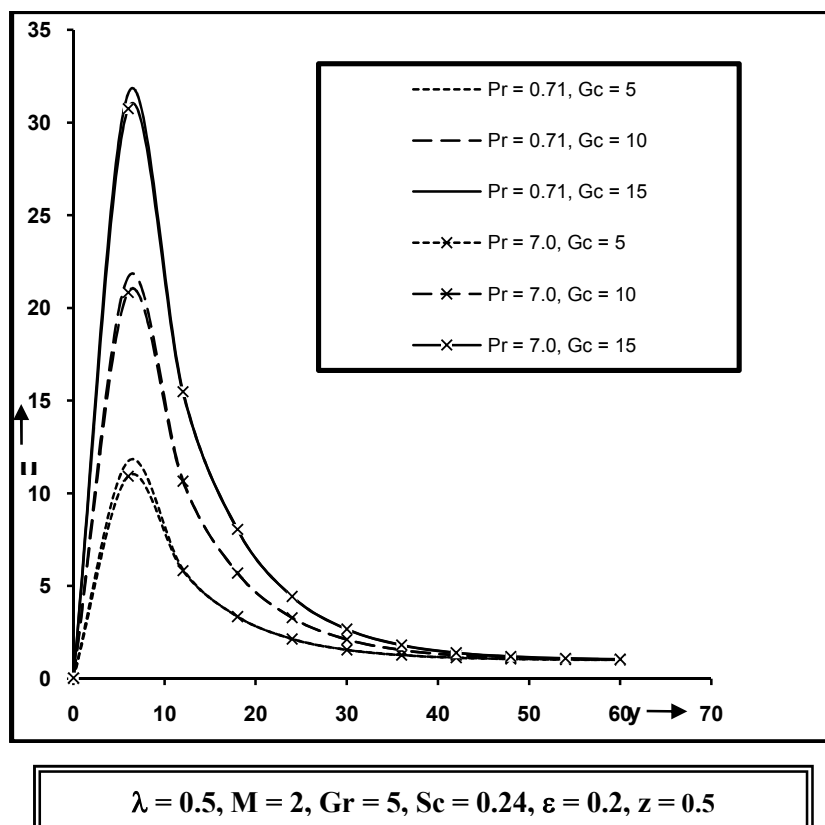
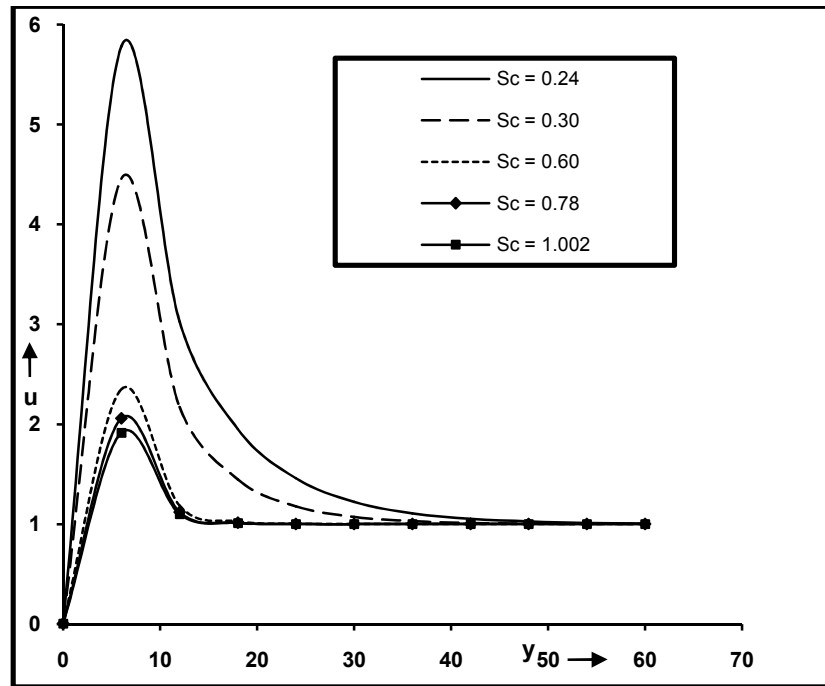
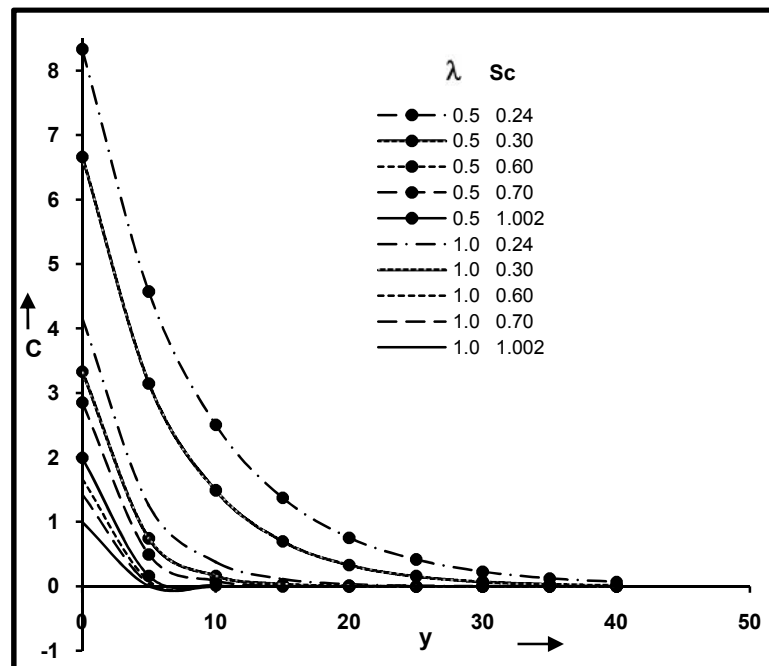


Fig. 1: Graph of u against y for different values of M and λ

Fig. 2: Graph of u against y for different values of Pr and Gr Fig. 3: Graph of u against y for different values of Pr and Gc

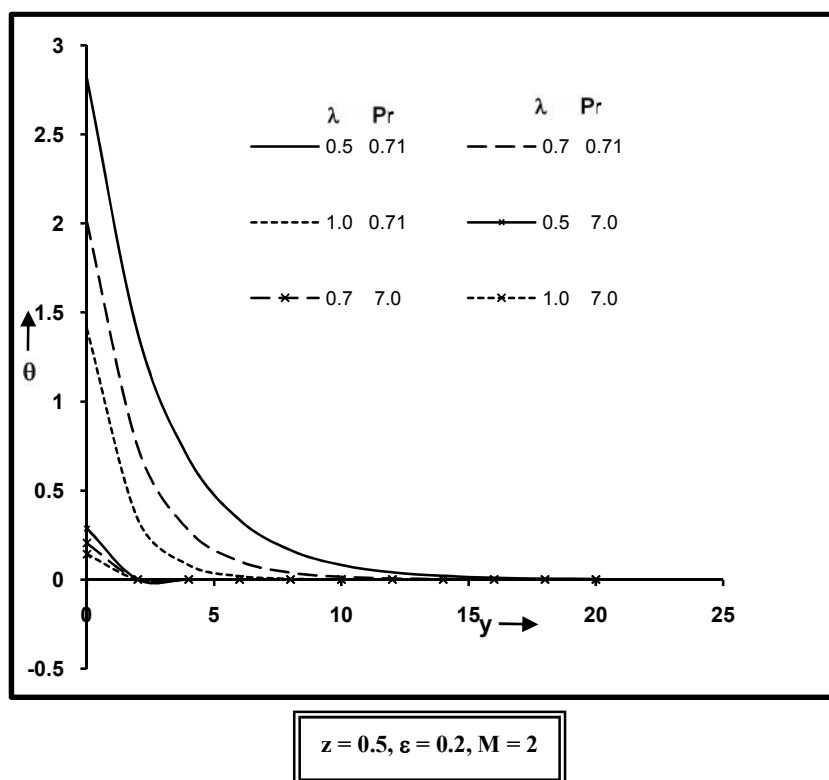
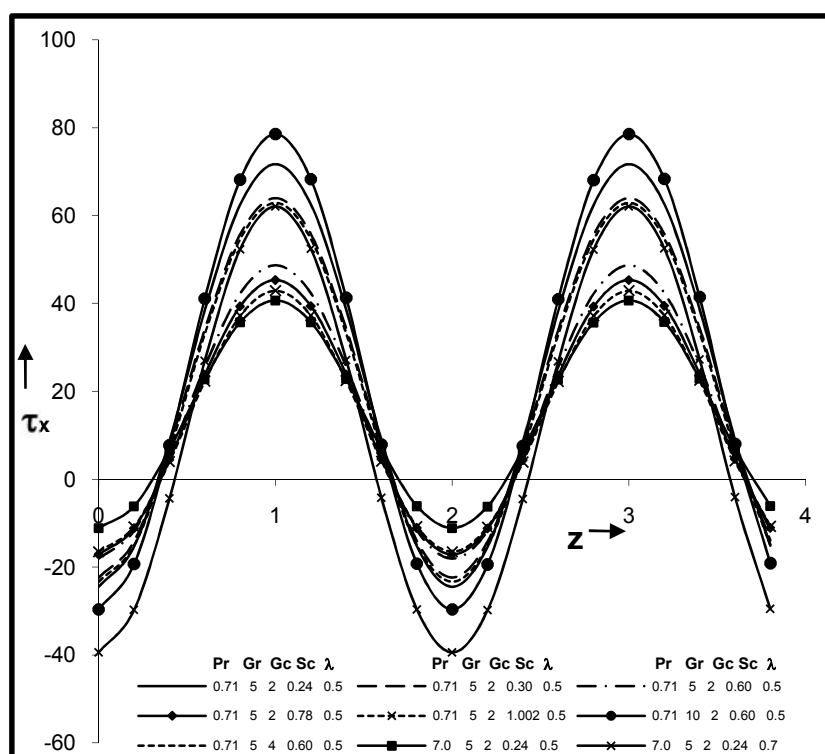


$$\lambda = 0.5, M = 2, Gr = 5, Gc = 2, \varepsilon = 0.2, z = 0.5, Pr = 0.71$$

Fig. 4: Graph of u against y for different values of Sc 

$$M = 2, Gr = 5, Gc = 2, \varepsilon = 0.2, z = 0.5, Pr = 0.71$$

Fig. 5: Graph of C against y for different values of λ and Sc

Fig. 6: Graph of θ against y for different values of Pr and λ Fig. 7: Graph of skin-friction for $\epsilon = 0.2$

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