

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination, December 2025

B. Tech., Odd Semester, 2025-26

Roll No.

AS1109: CALCULUS

Time: 2.5 hours

Max. Marks: 30

Instructions to students:

1. Do not write anything, other than your roll number, on the question paper.
2. Calculator is not allowed.
3. Mention all the assumptions for your answers clearly.

Q. 1.1 Find the value of b for which $1 + e^b + e^{2b} + e^{3b} + \dots = 3$.

1
(CO1)

Q. 1.2 Find a formula for the n th partial sum of the telescoping series,

$$\sum_{n=1}^{\infty} (\tan^{-1} n - \tan^{-1}(n+1))$$

2
(CO1)

and use it to determine if the series converges or diverges. If the series converges, find its sum.

Q. 2.1 Can a subset of $\mathbb{R}^2$ be neither close nor open? If yes, give an example of such subset. If no, explain why.

1
(CO2)

Q. 2.2 Find the range of the function $f(x, y) = \frac{1}{\sqrt{x^2 + y^2 - 36}}$.

1
(CO2)

Q. 2.3 The total resistance in a circuit containing two resistors x , and y is given by

$$R(x, y) = \frac{3xy}{3x + 3y + xy}$$

2
(CO2,3)

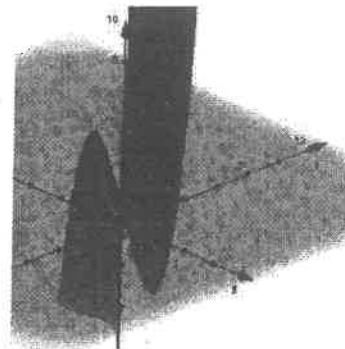
Suppose at a given time the x resistance is 1Ω , and the y resistance is 2Ω . Also, suppose the x resistance is changing at a rate of $2\Omega/\text{min}$, the y resistance is changing at the rate of $1\Omega/\text{min}$. Find the rate of change of the total resistance, $R(x, y)$ at that instant.

Q. 2.4 Find all the local maxima, local minima, and saddle points of the function

$$f(x, y) = x^3 + y^3 + 3x^2 - 3y^2.$$

The sketch of the surface $z = f(x, y)$ is shown in the accompanying figure.

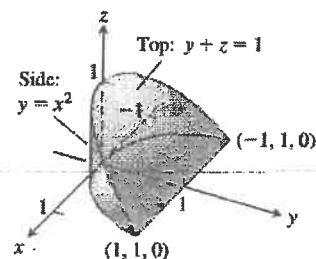
3
(CO2,3)



Q. 3.1 The volume of the solid shown in the accompanying figure is given by the triple integral

$$\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} dz dy dx.$$

Compute this triple integral to find the volume.



2
(CO2,4)

Q. 3.2 Sketch the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$ and hence, find its area.

3
(CO4)

Q. 3.3 Match the equation with the surface it defines. Also, identify each surface by type (paraboloid, ellipsoid, hyperboloid, etc.).

2
(CO2,4)

1. $9y^2 + z^2 = 16$

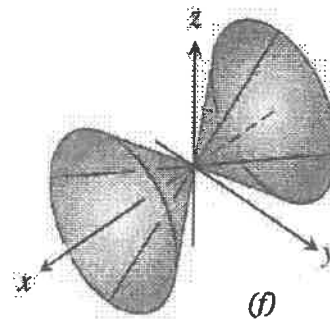
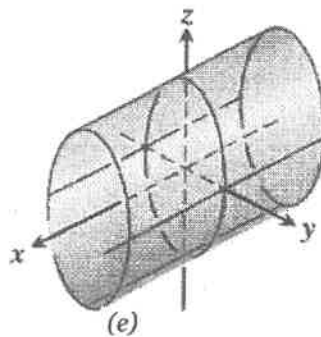
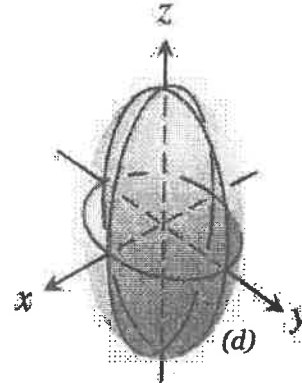
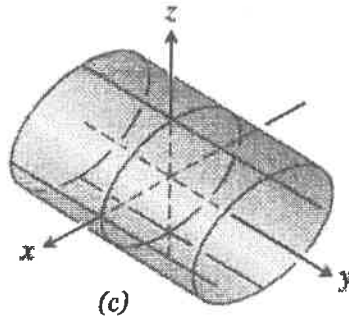
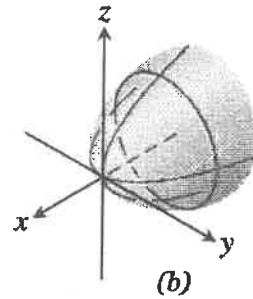
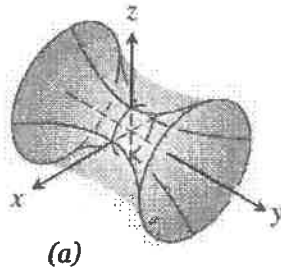
2. $x^2 + y^2 + 4z^2 = 10$

3. $y^2 + z^2 = x^2$

4. $x^2 + 2z^2 = 8$

5. $x = -(y^2 + z^2)$

6. $z^2 + x^2 - y^2 = 1$



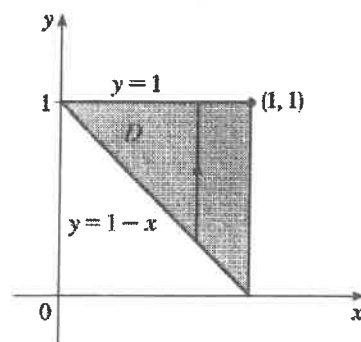
Q. 3.4 If an electric charge is distributed over a region D and the charge density (in coulombs per square meter) at a point (x, y) is given by $\sigma(x, y)$, then the total electric charge Q over the region D is

$$Q = \iint_D \sigma(x, y) dA.$$

Suppose the charge is distributed over the triangular region D shown in the accompanying figure, bounded by the lines $y = 1$, $x = 1$, and $x + y = 1$. The charge density at a point (x, y) in this region is

$$\sigma(x, y) = xy \text{ (in C/m}^2\text{)}.$$

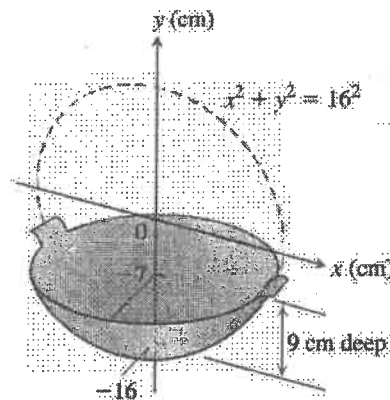
Find the total charge contained in the region D .



3
(CO3,4)

- Q. 3.5 You are designing a wok frying pan that will be shaped like a spherical bowl with handles. A bit of experimentation at home persuades you that you can get one that holds about 3 L if you make it 9 cm deep and give the sphere a radius of 16 cm.

Set up an integral to calculate the volume of the wok as a solid of revolution, as shown in the accompanying figure.



1
(CO2,4)

- Q. 4.1 Classify the following as a vector functions/vector fields or scalar functions/scalar fields.

- $\text{grad } f$
- Acceleration
- Electric potential
- Gravitational field
- Angular Momentum
- $\text{div } \vec{v}$
- Frequency
- Field of normal vectors of a surface

2
(CO5)

- Q. 4.2 Graph the velocity field, $\vec{v} = x\mathbf{i} - y\mathbf{j}$, of a fluid flow. By looking at the field lines, conclude whether the divergence of the velocity field, $\text{div } \vec{v}$, must be positive or negative, or may perhaps be zero.

2
(CO5)

- Q. 4.3 With respect to the right-handed coordinates, let $\vec{u} = [y^2, z^2, x^2]$ and $\vec{v} = [\sin y, \cos z, -\tan x]$, find $\vec{u} \times \text{curl } \vec{v}$.

2
(CO5)

- Q. 4.4 If $z(x, y) = 3000 - 2x^2 - 3y^2$ be the elevation (in meters) of a mountain above sea level. Find the direction of steepest ascent at the point $P(1, -2)$. Find the directional derivative of z at P in this direction.

3
(CO3.5)

:::END:::

