

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Regular & Backlog), November 2024

B. Tech. (CSE), Semester V, 2024-25

Roll No.....

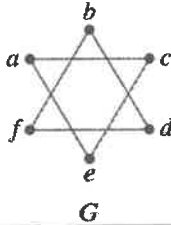
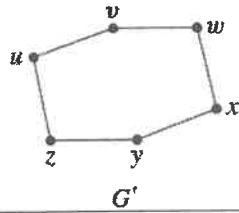
CS1142: Discrete Mathematics

Time: 3 hours

Max. Marks: 40

Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. Non programmable calculators are allowed.
3. Mention all the assumptions for your answers clearly.

Q. 1	a) Find all generators of the cyclic group $G = (\mathbb{Z}_6, +)$. b) Determine the generating function for the sequence of numbers $P(n, 0), P(n, 1), P(n, 2), \dots, P(n, n)$, where $P(n, r) = \frac{n!}{(n-r)!}$. c) How many cards must you pick from a standard 52-card deck to be sure of getting at least one red card? d) Consider the group G_1 of real numbers under addition and the group G_2 of positive real numbers under multiplication. Let $f: G_1 \rightarrow G_2$ be the function defined by $f(a) = 2^a$. Show that f is a group homomorphism.	1+1+1+1 (CO2,4,5)
Q. 2	Find the number of positive integers not exceeding 100 which are not divisible by 3, 5, or 7.	2 (CO5)
Q. 3	Draw two non-directed graphs with at least 4 vertices which has a) a Hamiltonian cycle but does not have an Eulerian circuit and b) an Eulerian circuit but does not have a Hamiltonian cycle.	2 (CO5)
Q. 4	Determine whether the given pair of graphs are isomorphic or not. Give proper justification for your answer.   G G'	2 (CO2,5)
Q. 5	Draw the complete graph, K_3 and write its adjacency matrix.	2 (CO5)
Q. 6	Let $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ be two trees where $ E_1 = 12$ and $ V_2 = 3 V_1 - 2$. Determine $ V_1 $, $ V_2 $ and $ E_2 $.	2 (CO5)
Q. 7	Verify the given logical equivalence using truth table, where p, q, r are propositional variables, $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$	2 (CO1)
Q. 8	Prove the given statement for all $n \geq 1$ by the Principle of Mathematical Induction, $\sum_{i=1}^n i(2^i) = 2 + (n-1)2^{n+1}.$	2 (CO1)
Q. 9	Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?	2 (CO5)

Q. 10	The set $G = \{1, -1, i, -i\}$ is a group under multiplication where $i^2 = -1$. a) Write the inverse of each of its elements. b) If $H = \{1, i\}$ be a nonempty subset of G . Prove or disprove: H is a subgroup of G .	2 (CO2,4)																																				
Q. 11	Let $G = \{q \in \mathbb{Q} \mid q \neq 1\}$ be a group with the binary operation $\blacksquare$ defined as $x \blacksquare y = x + y + xy$. Find the identity element and the inverse of each element of the group. Also, check whether it is an abelian group or not.	3 (CO2,4)																																				
Q. 12	A traveler in Europe wants to visit each of the cities shown on the map exactly once, starting and ending in Brussels. The distance (in kms) between each pair of cities is given in the table. Draw a graph for this problem and find a Hamiltonian cycle that minimizes the total distance travelled.  <table><tr><th></th><th>Berlin</th><th>Brussels</th><th>Düsseldorf</th><th>Luxembourg</th><th>Munich</th></tr><tr><th>Brussels</th><td>783</td><td></td><td></td><td></td><td></td></tr><tr><th>Düsseldorf</th><td>564</td><td>223</td><td></td><td></td><td></td></tr><tr><th>Luxembourg</th><td>764</td><td>219</td><td>224</td><td></td><td></td></tr><tr><th>Munich</th><td>585</td><td>771</td><td>613</td><td>517</td><td></td></tr><tr><th>Paris</th><td>1,057</td><td>308</td><td>497</td><td>375</td><td>832</td></tr></table>		Berlin	Brussels	Düsseldorf	Luxembourg	Munich	Brussels	783					Düsseldorf	564	223				Luxembourg	764	219	224			Munich	585	771	613	517		Paris	1,057	308	497	375	832	3 (CO5)
	Berlin	Brussels	Düsseldorf	Luxembourg	Munich																																	
Brussels	783																																					
Düsseldorf	564	223																																				
Luxembourg	764	219	224																																			
Munich	585	771	613	517																																		
Paris	1,057	308	497	375	832																																	
Q. 13	How many students are enrolled in a course either in calculus, discrete mathematics, data structures, or programming languages at a school if there are 507, 292, 312, and 344 students in these courses, respectively; 14 in both calculus and data structures; 213 in both calculus and programming languages; 211 in both discrete mathematics and data structures; 43 in both discrete mathematics and programming languages; and no student may take calculus and discrete mathematics, or data structures and programming languages, concurrently?	3 (CO2,5)																																				
Q. 14	There are available unlimited number of Rs.1, Rs. 5, Rs. 10, Rs. 20 and Rs.50 notes. Determine the number of different denominations of Rs. 500 if the number of 1 rupee notes are a multiple of 5, number of 5 rupee notes must not exceed 9, number of 10 rupee notes must not exceed 19 and one and only one 50 rupee note.	4 (CO5)																																				
Q. 15	Let Z^+ be the set of positive real numbers and R be a relation “is a multiple of” defined on Z^+ . Such that (Z^+, R) is a poset. Let $A = \{1, 2, 3, 4, 6, 8, 12, 48, 60, 96\}$ be a subset Z^+ . a) Check whether (A, R) is a poset or not. Justify properly. b) Draw the Hasse diagram of (A, R) . c) Check whether (A, R) is a lattices or not. Justify. d) Write the minimal and maximal elements of A . e) Identify the infimum and supremum of (A, R) .	5 (CO 3)																																				

All the best!