

JK LAKSHMIPAT UNIVERSITY, JAIPUR

INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Backlog), May 2019

B. Tech.(All Branches), Semester II

Roll No.....

COURSE CODE: MA 202

Time: 3 hours

Max. Marks: 100

COURSE TITLE: Differential Equations and Complex Analysis

Instructions to students:

1. Do not write anything other than your roll number on question paper.

Q.1A Objective type multiple choice questions (10 questions @ 1 mark)

- i. Which of the following is true for $f(z) = z^2$
(a) Analytic everywhere (b) nowhere analytic
(c) Analytic only at Origin (d) Analytic everywhere except at Origin
- ii. An integrating factor transforms first-order differential equation to
(a) Bernoulli's form (b) a homogeneous form
(c) a linear form (d) exact form
- iii. The particular solution of an ODE contains how many arbitrary constants
(a) depends upon the degree of the ODE (b) one
(c) depends upon the order of the ODE (d) zero
- iv. The steady state temperature distribution in a solid without a heat source can be expressed as:
(a) Poisson's equation (b) Laplace equation
(c) diffusion equation (d) wave equation
- v. The inverse Laplace transform of $\frac{1}{(2s-7)}$
(a) e^{-7t} (b) e^{7t} (c) $\frac{1}{2}e^{\frac{7t}{2}}$ (d) $\frac{1}{2}e^{\frac{-7t}{2}}$
- vi. The kernel of the Fourier cosine transform is
(a) e^{-st} (b) e^{-ist} (c) $\cos(sx)$ (d) $\sin(sx)$
- vii. If f is analytic at a point, then
(a) f is an entire function (b) f is harmonic function
(c) f is differentiable at that point (d) None of them
- viii. The Laplace transform of a function $f(t)$ is $F(s)$. Then, $L(e^{bt} f(t))$ is
(a) $F(s-b)$ (b) $F\left(\frac{s}{b}\right)$ (c) $F(s+b)$ (d) none of these
- ix. If $f(x) = e^x$, $0 < x < \pi$, then a_0 is
(a) $\frac{1}{\pi} \sin h\pi$ (b) $\frac{1}{2\pi} \sin h\pi$ (c) $\sin h\pi$ (d) $\frac{2}{\pi} \sin h\pi$
- x. The C.F. of the ODE $(D^2 - a^2)^2 y = x \cos x e^x$
(a) $(C_1 + C_2 x) \sin x + (C_3 - C_4 x) \cos x$ (b) $C_1 e^x + C_2 e^{-x}$
(c) $(C_1 + C_2 x) e^x + (C_3 + C_4 x) e^{-x}$ (d) $(C_1 \pm C_2 x) e^{\pm x}$

Q.2	Short answer type questions (4 questions @ 2.5 marks)	10
Q.2A	Solve the following PDE $p^2 z^2 + q^2 = p^2 q$.	
Q.2B	Find the Laplace transform of $e^{3t}(\cos at - \sin bt)$.	
Q.2C	Solve: $\frac{dy}{dx} + y = y^2$	
Q.2D	Find the Fourier cosine transform of e^{bt} .	
Q.3A	Evaluate the following integral using Cauchy integral formula $\int_C \frac{4}{(z-1)^3(z+2)} dz$ where c is the circle $ z = \frac{3}{2}$.	05
Q.3B	A resistance R and a capacitor C are connected in series with a voltage supply $E(t)$. develop a mathematical model for this circuit and find the current flowing through this circuit.	05
Q.4A	Determine the poles of the following function and residue at each pole $f(z) = \frac{e^{zt} dz}{z^2(z^2+2z+2)}$.	05
Q.4B	Apply method of variation of parameter $(D^2 + 1)y = \operatorname{cosec} x \cdot \cot x$	05
Q.5A	Find Fourier sine transform of as $f(x) = \begin{cases} x, & 0 < x \leq 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x \geq 2 \end{cases}$	05
Q.5B	Find the Fourier series expansion of $f(x) = x^3, -\pi \leq x \leq \pi$	05
Q.6A	Show that $x^2 - y^2 - y$ is harmonic function. Find the analytic function for which $x^2 - y^2 - y$ is real- part.	05
Q.7A	Find the Fourier transform of the function $f(x) = \begin{cases} 1 + \frac{x}{a} & \text{for } -a < x < 0 \\ 1 - \frac{x}{a} & \text{for } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$	05
Q.7B	A rod of length l with insulated sides is initially at a uniform temperature u . Its ends are suddenly cooled to 0°C and are kept at that temperature. Prove that the temperature function (x,t) is given by $u(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \cdot e^{-\frac{c^2 \pi^2 n^2 t}{l^2}}$ using separation of variables method/Fourier transform technique.	10
OR		
Q.7B	Solve the following initial value problem using Laplace transform $y'' + 7y' + 10y = 4e^{-t}, y(0) = 0, y'(0) = -1$	10
Q.8A	Evaluate the following integral using Cauchy integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where c is the circle $ z = \frac{3}{2}$.	10
Q.9A	Solve the following system of simultaneous differential equations using Laplace transform	10

$$2\frac{dx}{dt} + 4\frac{dy}{dt} + x = 3e^t$$

$$; x(0)=0, y(0)=0.$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 2y = e^{2t}$$

Q.9B Solve the differential equation: $(D^2 + 9)y = \sec 3x$.

10

OR

Q.9B Write the statement of Residue theorem. Then, apply this theorem to solve

10

$$\int_c \frac{1}{z(z+1)^2(z+3)} dz \text{ where } c \text{ is the circle } |z| = 2.$$

