

# JK LAKSHMIPAT UNIVERSITY, JAIPUR

## INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Backlog), November 2017

B. Tech (All Branches), Semester I

Roll No.....

COURSE CODE: MA 101

Time: 3 hours

Max. Marks: 100

COURSE NAME: Engineering Mathematics - I

### Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Assume suitable data wherever required and mention it clearly.

|       |                                                                                                                                                                                                                                                                                                                  |    |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Q.1A  | Objective type multiple choice questions (10 questions @ 1 mark)                                                                                                                                                                                                                                                 | 10 |
| i.    | Maxima and minima occur<br>(a) alternately (b) simultaneously (c) rarely (d) None of theses                                                                                                                                                                                                                      |    |
| ii.   | The line integral of function $\vec{F} = yz\hat{i}$ , in counterclockwise direction, along the circle $x^2 + y^2 = 1$ at $z=1$ is<br>(a) $\pi$ (b) $-\pi$ (c) $2\pi$ (d) None of theses                                                                                                                          |    |
| iii.  | If vectors A and B are irrotational, then<br>(a) $A \times B$ is irrotational (b) $A \cdot B$ is solenoidal (c) $A \times B$ is solenoidal (d) None of theses                                                                                                                                                    |    |
| iv.   | If $u=x+y$ and $v=xy$ then the value of $J\left(\frac{u,v}{x,y}\right)$ is<br>(a) $xy$ (b) $x+y$ (c) $x^2-y$ (d) $x-y$                                                                                                                                                                                           |    |
| v.    | The unit tangent vector of $\vec{r} = (2 + t^2)\hat{i} + (4t - 5)\hat{j} + (2t^2 - 6t)\hat{k}$ at $t = 2$ is<br>(a) $\frac{\hat{i}+2\hat{k}}{3}$ (b) $\frac{\hat{j}+2\hat{k}}{3}$ (c) $\frac{2\hat{i}+2\hat{j}+\hat{k}}{3}$ (d) $\frac{\hat{i}+2\hat{j}}{3}$                                                     |    |
| vi.   | What is the area of the region enclosed by $y^2=x$ and $y=x-2$ ?<br>(a) $11/2$ (b) $9/2$ (c) $5/2$ (d) $1/2$                                                                                                                                                                                                     |    |
| vii.  | The divergence of the vector field $x^2z\hat{i} + xy\hat{j} - yz^2\hat{k}$ at $(1,-1,1)$ is<br>(a) 0 (b) 3 (c) 5 (d) 6                                                                                                                                                                                           |    |
| viii. | Total derivative of the function 'xy'<br>(a) $dx \cdot dy$ (b) $dx + dy$ (c) $x dx + y dy$ (d) $x dy + y dx$                                                                                                                                                                                                     |    |
| ix.   | The value of $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$ is<br>(a) 2 (b) $\pi$ (c) $\pi/2$ (d) None of theses                                                                                                                                                                                                |    |
| x.    | The time 'T' of a complete oscillation of a simple pendulum of length 'L' is governed by the equation $T = 2\pi \sqrt{\frac{L}{g}}$ where $g$ is a constant. By what percentage should the length be changed in order to correct a loss of 2 minutes per day?<br>(a) -0.118% (b) -0.568% (c) -0.378% (d) -0.278% |    |
| Q.2   | Short answer type questions (4 questions @ 2.5 marks)                                                                                                                                                                                                                                                            | 10 |
| Q.2A  | Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point $P(2,1,3)$ in the direction of vector $\vec{a} = \hat{i} - 2\hat{k}$ .                                                                                                                                                          |    |

|      |                                                                                                                                                                                                                                                                                                                                           |    |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Q.2B | For $x = u, y = u \tan v, z = w$ , verify $\frac{\partial(x,y,z)}{\partial(u,v,w)} \cdot \frac{\partial(u,v,w)}{\partial(x,y,z)} = 1$ .                                                                                                                                                                                                   |    |
| Q.2C | The acceleration of a particle, which starts from rests, is $6t\hat{i} + 8t^3\hat{j} - 12t^2\hat{k}$ , at any instant $t \geq 0$ . Find its velocity at time 't'.                                                                                                                                                                         |    |
| Q.2D | Write the statement of Gauss divergence theorem and give one application where we can apply result of divergence theorem.                                                                                                                                                                                                                 |    |
| Q.3A | Trace the Folium of Descartes $x^3 + y^3 = 3axy$                                                                                                                                                                                                                                                                                          | 05 |
| Q.3B | If $x^x y^y z^z = c$ , show that at $x = y = z, \frac{\partial^2 z}{\partial x \partial y} = -(x \log x)^{-1}$ .                                                                                                                                                                                                                          | 05 |
| Q.4A | Find the volume of the solid generated by revolving the region bounded by the curves $y = 3 - x^2$ and $y = -1$ about the line $y = -1$ .                                                                                                                                                                                                 | 05 |
| Q.4B | Trace the Folium of Descartes $x^3 + y^3 = 3axy$                                                                                                                                                                                                                                                                                          | 05 |
| Q.5A | Discuss the extreme values of the function $x^3 + y^3 - 3axy$ .                                                                                                                                                                                                                                                                           | 05 |
| Q.5B | Evaluate $\iint_S \vec{F} \cdot \hat{n} dS$ where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant.                                                                                                                                      | 05 |
| Q.6A | State Euler's theorem then verify $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ if $u = \log_e \left( \frac{x^4 + y^4}{x + y} \right)$ .                                                                                                                                                                        | 05 |
| Q.7A | Find the asymptote parallel to the axis of 'x' of the curve $y^3 + x^2y + 2xy^2 - y + 2 = 0$ .                                                                                                                                                                                                                                            | 05 |
| Q.7B | Find the arc length of the curve $x = \log y$ between the ordinates $4/5$ and $5/4$ . Also, determine the unit normal vector for $r(t) = t\hat{i} + t^2\hat{j}$ when $t=1$ .                                                                                                                                                              | 10 |
| OR   |                                                                                                                                                                                                                                                                                                                                           |    |
| Q.7B | Prove that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find a scalar function $\phi(x,y,z)$ . "Does the value of a line integral $\int_C \vec{F} \cdot d\vec{r}$ change if we integrate from the same initial point to the same terminal point but along another path". Justify your answer. | 10 |
| Q.8A | Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ , where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ .                                                                                                                                                                                     | 10 |
| Q.9A | Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ using Lagrange's multipliers method.                                                                                                                                           | 10 |
| Q.9B | Discuss how curve tracing plays an important role for finding area and volume of any curve then trace the curve $x = a^3 \cos^3 t, y = a^3 \sin^3 t$ .                                                                                                                                                                                    | 10 |
| OR   |                                                                                                                                                                                                                                                                                                                                           |    |
| Q.9B | It is required to build a parabolic satellite dish, whose shape will be formed by rotating a parabolic curve $y = ax^2$ about the y-axis. If the dish is to be having a diameter of 10 ft and a maximum depth of 2 ft, find the value of 'a' and the surface area of the dish.                                                            | 10 |

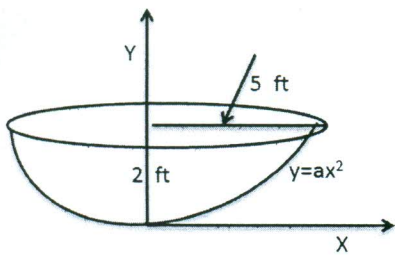


Fig. Parabolic satellite dish.