

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End-Term Examination, May 2025
 B. Tech., Even Semester, 2024-25

Roll No.....

AS1116: Random Variables and Stochastic Processes

Time: 2 hours

Max. Marks: 30

Instructions to students:

1. Do not write anything other than your roll number on the question paper.
2. You may use a non-programmable scientific calculator.
3. Specify if you find any ambiguity in the questions. Justify and solve the question after removing that ambiguity.

Q.1	Let $\{X_n, n \in \mathbb{Z}\}$ be a discrete-time random process, defined as $X_n = 2 \cos \left(\frac{n\pi}{8} + \Phi \right),$ Where $\Phi \sim \text{Uniform}(0, 2\pi)$ are two independent random variables. a) Find the mean function $\mu_X(n)$. b) Find the autocorrelation function $R_X(m, n)$. c) Is X_n a wide-sense stationary (WSS) process?	6.0 (CO2,4)
Q.2	Let $X(t)$ be a WSS process with autocorrelation function $R_X(\tau) = 1 + \delta(\tau).$ Assume that $X(t)$ is input to an LTI system with impulse response $h(t) = e^{-t}u(t).$ Let $Y(t)$ be the output. Find $S_X(f)$ and $H(f)$.	2.0 (CO2,4,5)
Q.3	The number of orders arriving at a service facility can be modeled by a Poisson process with intensity $\lambda = 10$ orders per hour. Find the probability that: a) There are no orders between 10:30 and 11. b) There are 3 orders between 10:30 and 11, and 7 orders between 11:30 and 12.	5.0 (CO2,4)
Q.4	Consider the Markov chain with three states $S = \{1, 2, 3\}$, that has the following state transition diagram. a) Find the state transition matrix for this chain. b) Identify the recurrent classes. Do you find any absorbing sets in these classes? Explain by defining both types of classes. c) Given that $P(X_0 = 1) = \frac{1}{3}$, find $P(X_0 = 1, X_1 = 2, X_2 = 3)$.	6.0 (CO1,2,3)

Q.5	50 students live in a dormitory. The parking lot has a capacity for 30 cars. Each student has a car with probability $\frac{1}{2}$, independently of other students. Use the central limit theorem (with continuity correction) to find the probability that there won't be enough parking spaces for all the cars?	5.0 (CO1,4)												
Q.6	Consider two random variables X and Y with joint probability mass function (PMF) given in the following table. <table border="1"> <tr> <th></th><th>$Y = 1$</th><th>$Y = 2$</th></tr> <tr> <th>$X = 1$</th><td>$\frac{1}{3}$</td><td>$\frac{1}{12}$</td></tr> <tr> <th>$X = 2$</th><td>$\frac{1}{6}$</td><td>0</td></tr> <tr> <th>$X = 4$</th><td>$\frac{1}{12}$</td><td>$\frac{1}{3}$</td></tr> </table> a) Find the conditional PMF of X given $Y = 1$. That is, find $P_{X Y}(x 1)$. b) Find $E[X Y = 1]$. c) Find $Var[X Y = 1]$.		$Y = 1$	$Y = 2$	$X = 1$	$\frac{1}{3}$	$\frac{1}{12}$	$X = 2$	$\frac{1}{6}$	0	$X = 4$	$\frac{1}{12}$	$\frac{1}{3}$	6.0 (CO1,3)
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