

JK Lakshmipat University Jaipur
INSTITUTE OF ENGINEERING AND TECHNOLOGY

End Term Examination (Backlog), December 2024

B. Tech. (All branches), Odd Semester, 2024-25

Roll No.....

AS1406 : Partial Differential Equation and Fourier Transform

Time: 2 hours

Max. Marks: 30

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Mention all the assumptions for your answers clearly.

Q. 1.1 Compute the Fourier transform of $f(x) = \begin{cases} \frac{\sin x}{x}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$ 2.0
(LO3)

Q. 1.2 Show that if $f(x)$ has a Fourier transform so does $f(x-a)$ and $F\{f(x-a)\} = e^{iaa}F\{f(x)\}$. 2.0
(LO3)

Q. 1.3 Solve the PDE $z = e^{ax+by} \cdot f(ax - by)$ 2.0
(LO1)

Q. 1.4 Find the partial differential equation of the family of spheres of radius 7 with centers on the plane $x - y = 0$. 2.0
(LO1)

Q. 2.1 Find the Fourier sine and Fourier cosine transform of the following functions. Also write the Fourier sine and cosine integrals for these functions. 3+3
(LO3)

(i) $f(t) = \begin{cases} k & \text{for } t < a \\ 0 & \text{for } t > a \end{cases}$

(ii) $f(t) = 2e^{-5t} + 5e^{-2t}$

Q. 2.2 Solve the following PDE 3+3
(LO1)

1. $x^2p + y^2q = z^2$

2. $y^2p - xyq = x(z - 2y)$

Q. 3.1 Find the temperature distribution $u(x, t)$ in semi-infinite bar with its end point and lateral surface insulated and with initial temperature distribution in the bar prescribed by $f(x)$. Deduce the solution using Fourier transform when 5.0
(LO2,3)

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

- Q. 3.2 Use the method of separation of variables to find the temperature $u(x,t)$ in a homogeneous bar of heat conducting material of length L cm with its ends kept at zero temperature and initial temperature given by $d x(L-X)/L^2$. 5.0
(LO1,2)
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all the best!